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MATH122 Enrol. No. ....

[ST]

END SEMESTER EXAMINATION : APRIL–MAY, 2017

APPLIED MATHEMATICS – II

Time : 3 Hrs. *Maximum Marks : 70*

Note: ***Attempt questions from all sections as directed.***

SECTION – A (30 Marks)

Attempt any **five** *questions out of* **six**.

Each question carries **06** *marks.*

1. Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x\log x$.

2. Form a partial differential equation by eliminating the arbitrary function from $\phi(x + y + z,\ x^2 + y^2 - z^2) = 0$.

3. Use Cauchy's Integral formula to evaluate

$$\int_C \frac{\cos^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz$$ where, C is the circle $|z| = 1$.

P.T.O.

4. Find the general solution of

$$x\left(z^2 - y^2\right)\frac{\partial z}{\partial x} + y\left(x^2 - z^2\right)\frac{\partial z}{\partial y} = z\left(y^2 - x^2\right).$$

5. Use Contour Integration to evaluate the real integral

$$\int_0^{\infty} \frac{dx}{\left(1 + x^2\right)^3}.$$

6. Solve $\frac{dx}{dt} + 2y + x = e^t$

$$\frac{dy}{dt} + 2x + y = 3e^t$$

SECTION – B **(20 Marks)**

Attempt any **two** *questions out of* **three**.

Each question carries **10** *marks.*

7. (a) Solve $(1+xy)x\,dy + (1-xy)y\,dx = 0$. (5)

(b) Determine the poles of the function

$$f(z) = \frac{z^2 - 2z}{(z+1)\left(z^2 + 1\right)}$$

and then find the residue at each pole. (5)

8. (a) Solve $(p^2 + q^2)y = qz$. (5)

(b) If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is constant. (5)

9. (a) Find the image of the infinite strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. (5)

(b) Solve $x^2 r - 3xys + 2y^2 t + px + 2qy = x + 2y$. (5)

SECTION - C **(20 Marks)**

(Compulsory)

10. (a) Solve $(D^2 + a^2)y = \tan(ax)$. (6)

(b) If $(a_1 + ib_1)(a_2 + ib_2)...(a_n + ib_n) = A + iB$, prove that

(i) $\left(a_1^2 + b_1^2\right)\left(a_2^2 + b_2^2\right)...\left(a_n^2 + b_n^2\right) = A^2 + B^2$

(ii) $\tan^{-1}\left(\frac{b_1}{a_1}\right) + \tan^{-1}\left(\frac{b_2}{a_2}\right) + ... + \tan^{-1}\left(\frac{b_n}{a_n}\right) = \tan^{-1}\left(\frac{B}{A}\right)$

(8)

P.T.O.

(c) Solve $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y$. (6)