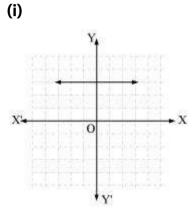
NCERT Solutions for Class 10 Maths Unit 2

Polynomials Class 10

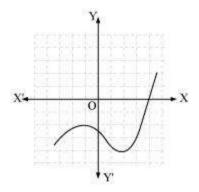
Unit 2 Polynomials Exercise 2.1, 2.2, 2.3 2.4, 2.4 Solutions

Exercise 2.1 : Solutions of Questions on Page Number : 28 Q1 :

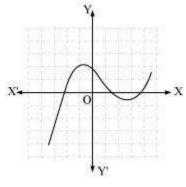
The graphs of y = p(x) are given in following figure, for some polynomials p(x). Find the number of zeroes of p(x), in each case.



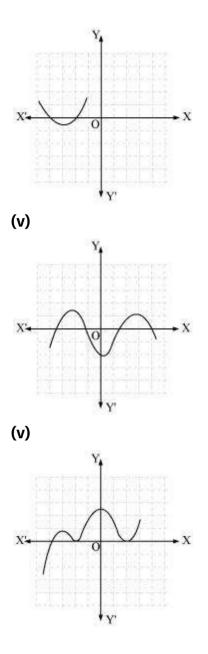












Answer:

(i) The number of zeroes is 0 as the graph does not cut the *x*-axis at any point.

- (ii) The number of zeroes is 1 as the graph intersects the *x*-axis at only 1 point.
- (iii) The number of zeroes is 3 as the graph intersects the *x*-axis at 3 points.
- (iv) The number of zeroes is 2 as the graph intersects the *x*-axis at 2 points.
- (v) The number of zeroes is 4 as the graph intersects the *x*-axis at 4 points.
- (vi) The number of zeroes is 3 as the graph intersects the *x*-axis at 3 points.

Q1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i)
$$x^{2}-2x-8$$
 (ii) $4s^{2}-4s+1$ (iii) $6x^{2}-3-7x$
(iv) $4u^{2}+8u$ (v) $t^{2}-15$ (vi) $3x^{2}-x-4$

Answer:

(i) $x^2 - 2x - 8 = (x - 4)(x + 2)$

The value of is zero when x - 4 = 0 or x + 2 = 0, i.e., when x = 4 or x = -2

Therefore, the zeroes of are 4 and - 2.

Sum of zeroes = $4-2=2=\frac{-(-2)}{1}=\frac{-(\text{Coefficient of }x)}{\text{Coefficient of }x^2}$ $= 4 \times (-2) = -8 = \frac{(-8)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$ (ii) $4s^2 - 4s + 1 = (2s - 1)^2$ The value of 4s2 - 4s + 1 is zero when 2s - 1 = $0, \tilde{s}i.\bar{e}.\bar{a}$ Therefore, the zeroes of $4s^2 - 4s + 12axed^2$ $\frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$ Sum of zeroes = $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$ (iii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3 = (3x+1)(2x-3)$ The value of $6x^2 - 3 - 7x$ is zero when 3x + 1 = 0 or 2x - 3 = 0, i.e., 3 = 0, i.e., 3 = 0Therefore, the zeroes of $6x^2 - 3 - 7x \frac{-1}{3}$ and $\frac{3}{2}$ $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$ Sum of zeroes

Product of zeroes $\frac{-1}{-3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iv) $4u^2 + 8u = 4u^2 + 8u + 0$ = 4u(u+2)

The value of $4u^2 + 8u$ is zero when 4u = 0 or u + 2 = 0, i.e., u = 0 or u = -2Therefore, the zeroes of $4u^2 + 8u$ are 0 and - 2.

Sum of zeroes = $0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes = $0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$

(v) $t^2 - 15$ = $t^2 - 0.t - 15$ = $(t - \sqrt{15})(t + \sqrt{15})$

The value of t2 - 15 is zero when $\sqrt{15} = 0$ or $t + \sqrt{15} = 0$, i.e., when

Q2 :

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i)	$\frac{1}{4}$, -1 (ii)	$\sqrt{2}, \frac{1}{3}$ (iii)	$0, \sqrt{5}$
(iv)	1,1 (v)	$-\frac{1}{4},\frac{1}{4}$ (vi)	4,1

Answer:

(i) $\frac{1}{4}, -1$

Let the polynomial $be^{ax^2} + bx + c$, and its zeroes be and β .

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$
$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$
If $a = 4$, then $b = -1$, $c = -4$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii)
$$\sqrt{2}, \frac{1}{3}$$

Let the polynomial $beax^2 + bx + c$, and its zeroes be and β .

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$
$$\alpha \beta = \frac{1}{3} = \frac{c}{a}$$
If $a = 3$, then $b = -3\sqrt{2}$, $c = 1$

Therefore, the quadratic polynomial is $3\frac{3}{2}\sqrt{2}x + 1$.

Let the polynomial bey and x its zeroes be and $\alpha = \beta$.

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$
$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$
If $a = 1$, then $b = 0$, $c = \sqrt{5}$

Therefore, the quadratic polynomial $\sqrt{5}$

Let the polynomial bey and its zeroes be

 α and β .

 $\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a}$ $\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$ If a = 1, then b = -1, c = 1

Therefore, the quadratic polynomial $\dot{s} - x + 1$.

 $(v) \quad -\frac{1}{4}, \frac{1}{4}$

Let the polynomial $beax^2 + bx + c$, and its zeroes be and

Exercise 2.3 2.4 : Solutions of Questions on Page Number : 36 Q1 :

Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$ (ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$ (iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

Answer:

(i)
$$p(x) = x^3 - 3x^2 + 5x - 3$$

 $q(x) = x^2 - 2$
 $x^2 - 2) \overline{x^3 - 3x^2 + 5x - 3}$
 $x^3 - 2x$
 $- +$
 $-3x^2 + 7x - 3$
 $-3x^2 + 6$
 $+ -$
 $7x - 9$

Quotient = x - 3

Remainder = 7x - 9

(ii)
$$p(x) = x^4 - 3x^2 + 4x + 5 = x^4 + 0 \cdot x^3 - 3x^2 + 4x + 5$$

 $q(x) = x^2 + 1 - x = x^2 - x + 1$

$$\begin{array}{r} x^{2} + x - 3 \\ x^{2} - x + 1 \hline x^{4} + 0.x^{3} - 3x^{2} + 4x + 5 \\ x^{4} - x^{3} + x^{2} \\ - + - \\ \hline x^{3} - 4x^{2} + 4x + 5 \\ x^{3} - x^{2} + x \\ - + - \\ \hline - 3x^{2} + 3x + 5 \\ - 3x^{2} + 3x - 3 \\ + - + \\ \hline 8 \end{array}$$

Quotient = $x^2 + x$ -

3 Remainder = 8

(iii)
$$p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$$

 $q(x) = 2 - x^2 = -x^2 + 2$
 $-x^2 + 2)$
 $x^4 + 0.x^2 - 5x + 6$
 $x^4 - 2x^2$
 $- +$
 $2x^2 - 5x + 6$
 $2x^2 - 4$
 $- +$
 $-5x + 10$

Quotient = $-x^2 - 2$ Remainder = -5x + 10

Q2 :

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

each case: (i) $2x^3 + x^2 - 5x + 2;$ $\frac{1}{2}, 1, -2$ (ii) $x^3 - 4x^2 + 5x - 2;$ 2,1,1

Answer:

(i)
$$p(x) = 2x^3 + x^2 - 5x + 2$$
.

Zeroes for this polynomial are $\frac{1}{2}$, 1, -2

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^{3} + \left(\frac{1}{2}\right)^{2} - 5\left(\frac{1}{2}\right) + 2$$
$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$
$$= 0$$
$$p(1) = 2 \times 1^{3} + 1^{2} - 5 \times 1 + 2$$
$$= 0$$
$$p(-2) = 2(-2)^{3} + (-2)^{2} - 5(-2) + 2$$
$$= -16 + 4 + 10 + 2 = 0$$
$$\frac{1}{2}$$

Therefore, 21, and - 2 are the zeroes of the given polynomial.

Comparing the given polynomial with, we obtain a = 2, b = 1, c = -5, d = 2

We can take
$$\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$$

 $\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$
 $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$
 $\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii)
$$p(x) = x^3 - 4x^2 + 5x - 2$$

Zeroes for this polynomial are 2, 1, 1.

$$p(2) = 2^{3} - 4(2^{2}) + 5(2) - 2$$

= 8 - 16 + 10 - 2 = 0
$$p(1) = 1^{3} - 4(1)^{2} + 5(1) - 2$$

= 1 - 4 + 5 - 2 = 0

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

$$ax^3 + bx^2 + cx + d$$

Comparing the given polynomial with , we obtain a = 1, b = -4, c = 5, d = -2. Verification of the relationship between zeroes and coefficient of the given polynomial

Sum of zeroes =
$$2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

Multiplication of zeroes taking two at a time = (2)(1) + (1)(1) + (2)(1) = 2 + 1 + 2 = $\frac{(5)}{5} = \frac{c}{a}$

Multiplication of zeroes =
$$2 \times 1 \times 1 = 2 \frac{-(-2)}{1} = \frac{-d}{a}$$

Hence, the relationship between the zeroes and the coefficients is verified.

Q3 :

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i)
$$t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$$

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$
(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Answer:

(i)
$$t^2 - 3$$
, $2t^4 + 3t^3 - 2t^2 - 9t - 12$
 $t^2 - 3 = t^2 + 0.t - 3$
 $t^2 + 0.t - 3$) $2t^2 + 3t + 4$
 $t^2 + 0.t - 3$) $2t^4 + 3t^3 - 2t^2 - 9t - 12$
 $2t^4 + 0.t^3 - 6t^2$
 $- - +$
 $3t^3 + 4t^2 - 9t - 12$
 $3t^3 + 0.t^2 - 9t$
 $- - +$
 $4t^2 + 0.t - 12$
 $4t^2 + 0.t - 12$
 $- - +$
 0

Since the remainder is 0,

Hence,
$$t^2 - 3$$
 is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.
(ii) $x^2 + 3x + 1$, $3x^4 + 5x^3 - 7x^2 + 2x + 2$
 $x^2 + 3x + 1$) $3x^4 + 5x^3 - 7x^2 + 2x + 2$
 $3x^4 + 9x^3 + 3x^2$
 $- - -$
 $-4x^3 - 10x^2 + 2x + 2$
 $-4x^3 - 12x^2 - 4x$
 $+ + +$
 $2x^2 + 6x + 2$
 0

Since the remainder is 0,

Hence, is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$. (iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$ $\frac{x^2 - 1}{x^3 - 3x + 1} x^5 - 4x^3 + x^2 + 3x + 1$ $x^5 - 3x^3 + x^2$ $\frac{-+--}{-x^3}$ +3x+1 $-x^3 + 3x - 1$ + - + 2

Since the remainder 0

Hence, is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

Q4:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, - 7, - 14 respectively.

Answer:

Let the polynomial $ax^3 + bx^2 + cx + d$ and the zeroes b \mathscr{C} , β , and γ . be It is given that

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$$
$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$
$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If a = 1, then b = -2, c = -7, d = 14

Hence, the polynomial is $^3 - 2x^2 - 7x + 14$.

Q5 :

Obtain all other zeroes of $x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer:

$$p(x) = 3x^{4} + 6x^{3} - 2x^{2} - 10x - 5$$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,
 $\therefore \left(x - \sqrt{\frac{5}{3}}\right) \left(x + \sqrt{\frac{5}{3}}\right) = \left(x^{2} - \frac{5}{3}\right)$ is a factor of $3x^{4} + 6x^{3} - 2x^{2} - 10x - 5$.
 $x^{2} - \frac{5}{3}$

Therefore, we divide the given polynomial $\hat{b}y = 3$.

$$x^{2} + 0.x - \frac{5}{3} \frac{3x^{2} + 6x + 3}{3x^{4} + 6x^{3} - 2x^{2} - 10x - 5}$$

$$3x^{4} + 0x^{3} - 5x^{2}$$

$$- - +$$

$$6x^{3} + 3x^{2} - 10x - 5$$

$$6x^{3} + 0x^{2} - 10x$$

$$- - +$$

$$3x^{2} + 0x - 5$$

$$3x^{2} + 0x - 5$$

$$- - +$$

$$0$$

$$3x^{4} + 6x^{3} - 2x^{2} - 10x - 5 = \left(x^{2} - \frac{5}{3}\right)\left(3x^{2} + 6x + 3x^{2} - 10x\right)$$

$$= 3\left(x^{2} - \frac{5}{3}\right)\left(x^{2} + 2x + 1x^{2} - 10x\right)$$

We factorize $x^2 + 2x + 1$

 $=(x+1)^2$

Therefore, its zero is given by x + 1 = 0

x = - 1

As it has the term $(x, the)^2$ efore, there will be 2 zeroes at x = -1.

Hence, the zeroes of the given polynomial are^{5} , $-\sqrt{\frac{5}{3}}$, -1 and - 1.

Q6:

On dividing by a polynomial g(x), the quotient and remainder were x - 2 and -2x + 4, respectively. Find g(x).

Answer:

 $p(x) = x^{3} - 3x^{2} + x + 2 \qquad \text{(Dividend)}$ g(x) = ? (Divisor) Quotient = (x - 2)

Remainder = (-2x + 4)Dividend = Divisor × Quotient + Remainder $x^{3} - 3x^{2} + x + 2 = g(x) \times (x - 2) + (-2x + 4)$ $x^{3} - 3x^{2} + x + 2 + 2x - 4 = g(x)(x - 2)$ $x^{3} - 3x^{2} + 3x - 2 = g(x)(x - 2)$

g(x) is the quotient when we divide $\begin{pmatrix} x^3 - 3x^2 + 3x - 2 \end{pmatrix}$ by $\begin{pmatrix} x - 2 \end{pmatrix}$

$$\begin{array}{r} x^{2} - x + 1 \\ x - 2 \overline{\smash{\big)}\ x^{3} - 3x^{2} + 3x - 2} \\ x^{3} - 2x^{2} \\ - x^{3} - 2x^{2} \\ - x^{2} + 3x - 2 \\ - x^{2} + 3x - 2 \\ - x^{2} + 2x \\ + - \\ \hline x - 2 \\ x - 2 \\ - + \\ \hline 0 \\ \hline \vdots g(x) = (x^{2} - x + 1) \end{array}$$

Q7 :

Give examples of polynomial p(x), g(x), q(x) and r(x), which satisfy the division algorithm and

(i) deg p(x) = deg q(x)(ii) deg q(x) = deg r(x)

(iii) deg r(x) = 0

Answer:

According to the division algorithm, if p(x) and g(x) are two polynomials with $g(x) \neq 0$, then we can find polynomials q(x) and r(x) such that $p(x) = g(x) \times q(x) + r(x)$, where r(x) = 0 or degree of r(x) < degree of g(x) Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x) = \deg q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division $q_x by_+ 2x + 2$

```
Here, p(x) = 6x^2 + 2x + 2
g(x) = 2
q(x) = \text{and } r(x) = 0
Degree of p(x) and q(x) is the same i.e., 2.
Checking for division algorithm,
p(x) = g(x) \times q(x) + r(x)
= 20
6x^2 + 2x + 2 3x^2 + x + 1
= 6x^2 + 2x + 2
Thus, the division algorithm is satisfied.
(ii) deg q(x) = \deg r(x)
Let us assume the division of x3 + x by x2,
Here, p(x) = x3 + x
g(x) = x^2
q(x) = x and r(x) = x
Clearly, the degree of q(x) and r(x) is the same i.e., 1.
Checking for division algorithm,
p(x) = g(x) \times q(x) + r(x)
x3 + x = (x2) \times x + x
x3 + x = x3 + x
Thus, the division algorithm is satisfied.
(iii)deg r(x) = 0
Degree of remainder will be 0 when remainder comes to a constant.
Let us assume the division of x3 + 1by x2.
Here, p(x) = x3 + 1
g(x) = x^2
q(x) = x and r(x) = 1
Clearly, the degree of r(x) is 0.
```

Checking for division algorithm,

 $p(x) = g(x) \times q(x) + r(x)$ x3 + 1 = (x2) × x + 1 x3 + 1 = x3 + 1 Thus, the division algorithm is satisfied.

Exercise 2.4 : Solutions of Questions on Page Number : 37 Q1 :

If the zeroes of polynomial $3a^{2} + x + 1$ are a - b, a, a + b, find a and b.

Answer:

 $p(x) = x^3 - 3x^2 + x + 1$ Zeroes are *a* - *b*, *a* + *a* + *b*

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain p = 1, q = -3, r = 1, t = 1

Sum of zeroes = a - b + a + a + b

$$\frac{-q}{p} = 3a$$
$$\frac{-(-3)}{1} = 3a$$
$$3 = 3a$$
$$a = 1$$

The zeroes are b, 1, 1+b.

Multiplication of zeroes = 1(1-b)(1+b)

$$\frac{-t}{p} = 1 - b^{2}$$

$$\frac{-1}{1} = 1 - b^{2}$$

$$1 - b^{2} = -1$$

$$1 + 1 = b^{2}$$

$$b = \pm \sqrt{2}$$
Hence, $\alpha = 1$ and $b = \sqrt{2}$ or $-\sqrt{2}$.

Q2 :

]It two zeroes of the polynomial are, find other zeroes. $2\pm\sqrt{3}$

Answer:

Given that 2 + and 2 are zeroes of the given polynomial.

Therefore, $(\pm x^2 \pm x\sqrt{3})$ $(x = 3 + \sqrt{3})$

= $x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $\frac{1}{2}x_{26x}^{4x} \frac{1}{26x} + 138x - 35$

$$\begin{array}{r} x^2 - 2x - 35 \\ x^2 - 4x + 1 \overline{\smash{\big)}} x^4 - 6x^3 - 26x^2 + 138x - 35 \\ x^4 - 4x^3 + x^2 \\ - + - \\ - 2x^3 - 27x^2 + 138x - 35 \\ - 2x^3 + 8x^2 - 2x \\ + - + \\ - 35x^2 + 140x - 35 \\ - 35x^2 + 140x - 35 \\ + - + \\ 0 \end{array}$$

Clearly, $x^4 - 6x^3 - 26x^2 + 138x - 35$ $(x^2 - 4x + 1)(x^2 - 2x - 35)$

It can be observed that is also a factor of the given polynomial. And = Therefore, the value of the polynomial is also zero when or Or x = 7 or - 5 Hence, 7 and - 5 are also zeroes of this polynomial. x-7=0 x+5=0

Q3:

If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be x + a, find k and a.

Answer:

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend - Remainder = Divisor × Quotient

 $x^{4}-6x^{3}+16x^{2}-25x+10-x-a=x^{4}-6x^{3}+16x^{2}-26x+10-a$ will be perfectly divisible by $x^2 - 2x + k$. Let us divide $x^4 - 6x^3 + 16x^2 - 26x + 10 - a_{bv} x^2 - 2x + k$ $x^{2} - 4x + (8 - k)$ $x^{2} - 2x + k x^{4} - 6x^{3} + 16x^{2} - 26x + 10 - a$ $x^{4} - 2x^{3} + 16x^{2} - 26x + 10 - a$ $x^4 - 2x^3 + kx^2$ $\frac{-+--}{-4x^3+(16-k)x^2-26x}$ $-4x^3 + 8x^2 - 4kx$ $\frac{+ - +}{(8-k)x^2 - (26-4k)x + 10-a}$ $(8-k)x^2 - (16-2k)x + (8k-k^2)$ - + - $(-10+2k)x+(10-a-8k+k^2)$ It can be observed that $(-10+2k)x + (10-a-8k+k^2)$ will be 0. Therefore, $(-10+2k)_{=0} = 0$ and $(10-a-8k+k^2)_{=0}$ For $\left(-10+2k\right)_{=0}$ 2k = 10And thus, k = 5 $\int_{\text{For}} (10 - a - 8k + k^2)_{= 0}$ $10 - a - 8 \times 5 + 25 = 0$ 10 - a - 40 + 25 = 0

- 5 - *a* = 0 Therefore, *a* = - 5 Hence, *k* = 5 and *a* = - 5