Bihar Board Class 12 Mathematics Question Paper 2016

INTERMEDIATE EXAMINATION - 2016

(ANNUAL)

MATHEMATICS

Time- $3\frac{1}{4}$ **Hours**

Full Marks: 100

Instruction for the candidates:

1) Candidates are required to give their answers in their own words as far as practicable.
2) Figures in the right hand margin indicate full marks.
3) 15Minutes of extra time has been allotted for the candidates to read the questions carefully.
4) This question paper is divided into two **section A** and **section B.**
5) In **section-A,** there are **40 objective type questions** which are compulsory, each carry **1 mark.** Darken the circle with blue/black ball pen against the correct option on **OMR** answer Sheet provided to you. **Do not use whitener/Liquid/Blade/Nall etc. on OMR Sheet; otherwise the result will be invalid.**
6) In **Section-B,** there **are 25 short answer** type question (each carrying **2 marks**) out of which **any 15** questions are to be answered. Apart this, there are **8 Long Answer** Type question (Each Carrying **5 Marks**), Out of which **any 4** questions to be answered.
7) Use of any electronic appliances is strictly prohibited.

Section-I : (Objective Type)

For the following Q. Nos. 1 to 40 there is only correct answer against each question. Mark the correct option on the answer sheet.

1. $f : A \to B$ **will be an onto function if**

(A) $f(A) \subset B$ **(B)** $f : A \to B$

(B) $f(A) \supset B$ **(D)** $f(A) \neq B$

Sol. **Correct option is (A)**

The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ **is**

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{3\pi}{4}$

Sol.

Correct option is (C).

2. $\tan^{-1} x + \cot^{-1} x =$

(A) π

(B) $\frac{\pi}{2}$

(C) $\frac{2\pi}{3}$

(d) $\frac{3\pi}{4}$

Sol.

Correct option is (B).

3. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix}$

(A) $(a+b)(b+c)(c+a)$

(B) $(a+b)(b-c)(c-a)$

(C) $(a-b)(b-c)(c+a)$

(D) $(a-b)(b-c)(c-a)$

Sol.

Correct option is (D).

4. $A = \begin{bmatrix} 3 & 6 \\ 5 & -4 \end{bmatrix}, B = \begin{bmatrix} 7 & 8 \\ 5 & 6 \end{bmatrix} \Rightarrow 2A + 3B =$

(A) $\begin{bmatrix} 27 & 24 \\ 25 & 10 \end{bmatrix}$

(B) $\begin{bmatrix} 27 & 36 \\ 25 & 10 \end{bmatrix}$

(C) $\begin{bmatrix} 27 & 36 \\ 25 & 15 \end{bmatrix}$ **(D)** $\begin{bmatrix} 27 & 36 \\ 35 & 10 \end{bmatrix}$

Sol.

Correct option is (B).

5. $\frac{d}{dx}\left(\cos^{-1} x\right) =$

(A) $\frac{1}{2\sqrt{1-x^2}}$ **(B)** $\sqrt{1-x^2}$

(C) $\frac{-1}{\sqrt{1-x^2}}$ **(D)** $\frac{1}{\sqrt{1-x^2}}$

Sol.

Correct option is (C).

6. $\frac{d}{dx}\left(\tan^{-1} x + \cot^{-1} x\right) =$

(A) $\frac{2}{1+x^2}$ **(B)** 0

(C) 1 **(D)** 2

Sol.

Correct option is (C).

7. If $y = \cos\left(\log x\right)$**, then** $\frac{dy}{dx}$

(A) $-\sin\left(\log x\right)$ **(B)** $\frac{-\sin\left(\log x\right)}{x}$

(C) $\frac{\cos\left(\log x\right)}{x}$ **(D)** $-\sin\left(\log x\right)\log x$

Sol.

Correct option is (B).

8. If $y = x^3$, **then** $\frac{d^2y}{dx^2} =$

(A) $3x^2$

(B) $6x$

(C) 6

(D) 0

Sol.

Correct option is(B).

9. $\int x^8 dx$

(A) $8x^2 + k$

(B) $\frac{x^8}{8} + k$

(C) $x^9 + k$

(D) $\frac{x^9}{9} + k$

Sol.

Correct option is (D).

10. The integration of O with respect to x is :

(A) 0

(B) k

(C) $x + k$

(D) $x^2 + k$

Sol.

Correct option is (B).

11. $\int \frac{dx}{1 - \sin x} =$

(A) $\tan x - \sec x + k$

(B) $\tan x + \sec x + k$

(C) $\tan^2 x + \sec^2 x + k$

(D) $2(\tan x - \sec x) + k$

Sol.

Correct option is (B).

12. $\int_{b}^{a} x^2 dx$

(A) $\frac{a^3 - a^3}{3}$

(B) $\frac{a^3 - b^3}{3}$

(C) $\frac{a^2 - b^2}{2}$

(D) $\frac{b^2 - a^2}{2}$

Sol.

Correct option is (B).

13. The solution of the different equation $\frac{dy}{dx} = e^{x+y}$ **is**

(A) $e^x + e^{-y} + k = 0$

(B) $e^{2x} = ke^y$

(C) $e^x = ke^{2y}$

(D) $e^x = ke^y$

Sol.

Correct option is (A).

14. The integrating factor of the linear differential equation $\frac{dy}{dx} + Py = 0$

(A) $\int_{e} Pdy$

(B) $\int_{e} Qdx$

(C) $\int_{e} Qdy$

(D) $\int_{e} Pdx$

Sol.

Correct option is (A).

15. The order of the differental equation $\left(\frac{dy}{dx}\right)^2 + y = x$ **is**

(A) 0 **(B)** 1

(C) 2 **(D)** 3

Sol.

Correct option is (B).

16. The degree of the equation $\left(\frac{d^2y}{dx^2}\right)^2 - x\left(\frac{dy}{dx}\right)^3 = y^3$ **is**

(A) 0 **(B)** 1

(C) 2 **(D)** 3

Sol.

Correct option is(C).

17. The position vector of the point (x, y, z) is

(A) $x\vec{i} - y\vec{j} - x\vec{k}$ **(B)** $x\vec{i} + y\vec{j} - z\vec{k}$

(C) $x\vec{i} - y\vec{j} - z\vec{k}$ **(D)** $x\vec{i} + y\vec{j} + z\vec{k}$

Sol.

Correct option is(D).

18. $\left|-\vec{i} + 2\vec{j} - 3\vec{k}\right| =$

(A) $\sqrt{15}$ **(B)** $\sqrt{3}$

(C) 2 **(D)** $\sqrt{14}$

Sol.

Correct option is (D).

19. If the position vectors of the points A and B be respectively (1, 2, 3) and (-3, -4, 0) then $\overrightarrow{AB} =$

(A) $4\vec{i}+6\vec{j}+3\vec{k}$ **(B)** $-4\vec{i}-6\vec{j}-3\vec{k}$

(C) $-3\vec{i}-8\vec{k}$ **(D)** $-3\vec{i}-8\vec{j}$

Sol.

Correct option is (D).

21.If $\vec{a}=3\vec{i}+2\vec{j}+\vec{k}, \vec{b}=4\vec{i}-5\vec{j}+3\vec{k},$ **then** $\vec{a}\cdot\vec{b}$

(A) 2 **(B)** 3

(C) 5 **(D)** 7

Sol.

Correct option is (C)

22. If $\vec{a}$ **and** $\vec{b}$ **are perpendicular to each other then**

(A) $\vec{a}\cdot\vec{b}=0$ **(B)** $\vec{a}\times\vec{b}=\vec{0}$

(C) $\vec{a}+\vec{b}=\vec{0}$ **(D)** $\vec{a}-\vec{b}=0$

Sol.

Correct option is (A).

23. $\vec{a}\times\vec{a}=$

(A) 0 **(B)** 1

(C) a^2 **(D) a**

Sol.

Correct option is (B).

24. $\vec{i}\times\vec{j}=$

(A) 0 **(B)** 1

(C) $\vec{k}$ **(D)** $-\vec{k}$

Sol.

Correct option is (C).

25. $\vec{k} \cdot \vec{k} =$

(A) 0 **(B)** 1

(C) $\vec{i}$ **(D)** $\vec{j}$

Sol.

Correct option is (B).

26. The direction cosines of the x-axis are

(A) (0,0,0) **(B) (1,0,0)**

(C) (0,1,0) **(D) (0,0,1)**

Sol.

Correct option is (B).

27. l, m, n, are the direction cosines of a straight line them

(A) $l^2 + m^2 - n^2 = 1$ **(B)** $l^2 - m^2 + n^2 = 1$

(C) $l^2 - m^2 - n^2 - 1$ **(D)** $l^2 + m^2 + n^2 = 1$

Sol.

Correct option is (D).

28. The distance between the points (4,3,7) and (1,-1,-5) is

(A) 7 **(B) 12**

(C) 13 **(D) 25**

Sol.

Correct option is (C).

29. The direction between the points (4,3,7) and (1,-1,-5) is

(A) $\frac{1}{\sqrt{35}}, \frac{3}{\sqrt{35}}, \frac{5}{\sqrt{35}}$ **(B)** $\frac{1}{9}, \frac{1}{3}, \frac{5}{9}$

(C) $\frac{3}{\sqrt{35}}, \frac{5}{\sqrt{35}}, \frac{1}{\sqrt{35}}$ **(D)** $\frac{5}{\sqrt{35}}, \frac{3}{\sqrt{35}}, \frac{1}{\sqrt{35}}$

Sol.

Correct option is (A).

30. The direction ratios of two straight lines are l,m,n and l_1, m_1, n_1**.The lines will be perpendicular to each other if.**

(A) $\frac{l}{l_1} = \frac{m}{m_1} = \frac{n}{n_1}$ **(B)** $\frac{l}{l_1} + \frac{m}{m_1} + \frac{n}{n_1} = 0$

(C) $ll_1 + mm_1 + nn_1 = 0$ **(D)** $ll_1 + mm_1 + nn_1 = 1$

Sol.

Correct option is (C).

31. A line passing through (2, -1, 3) and its direction ratios are 3, -1, 2. The equation of the line is

(A) $\frac{x+2}{3} = \frac{y-1}{-1} = \frac{z+1}{2}$ **(B)** $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$

(C) $\frac{x-3}{2} = \frac{y+1}{-1} = \frac{z-2}{3}$ **(D)** $\frac{x-3}{2} = \frac{y+1}{-1} = \frac{z-1}{3}$

Sol.

Correct option is (B).

32. The lines $\frac{x-1}{l}=\frac{y+2}{m}=\frac{z-4}{n}$ **and** $\frac{x+3}{2}=\frac{y-4}{3}=\frac{z}{6}$ **are parallel is each other if**

(A) $2l=3m=n$

(B) $3l=2m=n$

(C) $2l+3m+6n=0$

(D) $lmn=36$

Sol.

Correct option is (D).

33. The length of the perpendicular from the point (0,-1,3) to the plane $2x+y-2z+1=0$ **his.**

(A) 0

(B) $2\sqrt{3}$

(C) $\frac{2}{3}$

(D) 2

Sol.

Correct option is (D).

34. If $P(A)=\frac{3}{8},P(B)=\frac{1}{2},P(A\cap B)=\frac{1}{4}$**, then** $P(A/B)=$

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{3}{8}$

Sol.

Correct option is (D).

35.If A and B are two independent events, then

(A) $P(AB') = P(A)P(B)$ **(B)** $P(AB') = P(A)P(B')$

(C) $P(AB') = P(A') + P(B)$ **(D)** $P(AB') = P(A) + P(B')$

Sol.

Correct option is (A).

36. The matrix $\begin{bmatrix} 3 & 5 \\ 2 & \text{k} \end{bmatrix}$ **has no inverse if the value of k is**

(A) 0 **(B) 5**

(C) $\frac{10}{3}$ **(D)** $\frac{4}{9}$

Sol.

Correct option is (A).

37. $\begin{vmatrix} 2 & 3 & 5 \\ 0 & 4 & 7 \\ 0 & 0 & 5 \end{vmatrix}$

(A) 40 **(B) 0**

(C) 3 **(D) 25**

Sol.

Correct option is (A).

38. $\tan^{-1}\frac{2x}{1-x^2}$

(A) $2\sin^{-1}x$ **(B)** $\sin^{-1}2x$

(C) $\tan^{-1} 2x$ **(D)** $2\tan^{-1} x$

Sol.

Correct option is (D).

39. $\int \frac{-1}{1+x^2} dx =$

(A) $\tan^{-1} x + k$ **(B)** $\sec^{-1} x + k$

(C) $\cos ec^{-1} x + k$ **(D)** $\cot^{-1} x + k$

Sol.

Correct option is (D).

40. $\int \frac{dx}{x^2 + a^2} =$

(A) $\frac{1}{a}\tan^{-1}\frac{x}{a} + k$ **(B)** $\frac{1}{a}\tan^{-1}(x+a) + k$

(C) $\sin^{-1}\frac{x}{a} + k$ **(D)** $\cos^{-1}\frac{x}{a} + k$

Sol.

Correct option is (A).

Section – II : (Non Objective Type)

Question Nos. 1 to 8 are of short answer type. Each question carries 4 marks. [8*4]

Short Answer Type Questions

1.Prove that $4\left(\cos^{-1} 3 \pm \cos e^{-1}\sqrt{5}\right) = \pi$

Sol.

Prove the expression,

$$\cot^{-1} = \frac{1}{\tan^{-1}}$$

$$\tan^{-1} = \frac{1}{3}$$

$$\cos ec^{-1} = \frac{1}{\sin^{-1}} = \frac{1}{\sqrt{5}}$$

$$P = 1 \quad \text{h=}\sqrt{5}$$

$$b = \sqrt{(h)^2 - (p)^2}$$

$$b = \sqrt{\left(\sqrt{5}\right)^2 - (1)^2} = \sqrt{4} = 2$$

$now,$

$$4\left(\cot^{-1} 3 + \cos ec^{-1}\sqrt{5}\right)$$

$$4\left(\tan^{-1}\frac{1}{3} + \tan\frac{1}{2}\right)$$

$$\tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

$$4\tan^{-1}\left(\frac{1/3+1/2}{1-1/2\times 1/3}\right)$$

$$4\tan^{-1}\left(\frac{5/6}{1-1/6}\right)$$

$$4\tan^{-1}\left(\frac{5/6}{5/6}\right)$$

$$4\tan^{-1}(1)$$

$$4\tan^{-1}\frac{\pi}{4} = \pi \ \left(R.H.S\right)$$

Hence, prove it.

2.If $A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ **, then find the value of** $A^2 + 3A + 2I$

Sol.

Simplify the expression,

$$A^2 = A \cdot A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$$A^2 = \begin{vmatrix} 1 \cdot 1 + 2 \cdot 3 & 1 \cdot 2 + 2 \cdot 4 \\ 3 \cdot 1 + 4 \cdot 3 & 3 \cdot 2 + 4 \cdot 4 \end{vmatrix} = \begin{vmatrix} 1+6 & 2+8 \\ 3+12 & 6+16 \end{vmatrix}$$

$$A^2 = \begin{vmatrix} 7 & 10 \\ 15 & 22 \end{vmatrix}$$

$$A^2 + 3A + 2I = \begin{vmatrix} 7 & 10 \\ 15 & 22 \end{vmatrix} + \begin{vmatrix} 3 & 6 \\ 9 & 12 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}$$

$$A^2 + 3A + 2I = \begin{vmatrix} 12 & 16 \\ 24 & 36 \end{vmatrix}$$

Hence, the solution is $A^2 + 3A + 2I = \begin{vmatrix} 12 & 16 \\ 24 & 36 \end{vmatrix}$.

3. Evaluate $\begin{bmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{bmatrix}$

Sol.

Simplify the expression,

$$\Delta = xyz\begin{bmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{bmatrix}$$

Applying $R_1 \to R_2 - R_3$, we get

$$\Delta = xyz\begin{bmatrix} 1\text{-}1 & x-y & x^2-y^2 \\ 1\text{-}1 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{bmatrix}$$

$$\Delta = xyz\begin{bmatrix} 0 & x-y & (x+y)(x-y) \\ 1\text{-}1 & y-z & (y+z)(y-z) \\ 1 & z & z^2 \end{bmatrix}$$

$$\Delta = xyz(x-y)(y-z)\begin{bmatrix} 0 & 1 & x+y \\ 0 & 1 & y+z \\ 1 & 1 & z^2 \end{bmatrix}$$

Now, expand, with respect to R_3, we get

$$\Delta = xyz(x-y)(y-z)\begin{bmatrix} 1 & x+y \\ 1 & y+z \end{bmatrix}$$

$$\Delta = xyz(x-y)(y-z)(y+z-x-y) \Rightarrow \Delta = xyz(x-y)(y-z)(y+z-x-y)$$

4. Solve for x: $\tan^{-1} 2x + \tan^{-1} 3x = \dfrac{\pi}{4}$

Sol.

Simplify the expression,

$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

$\tan^{-1} \frac{x+y}{1-xy}, xy < 1$

$\tan^{-1} \frac{2x+3x}{1-2x \cdot 3x} = 1$

$\frac{5x}{1-6x} = 1$

$5x = 1 - 6x$

$5x + 6x = 1$

$11x = 1$

$x = \frac{1}{11}$

Hence, the value of $x = \frac{1}{11}$.

5. If $y = \sin[\cos\left(\tan\left(\sin^{-1} x\right)\right)$. **then find** $\frac{dy}{dx}$

Sol.

Simplify the expression,

Given $y = \sin[\cos\left(\tan\left(\sin^{-1} x\right)\right)$.

$\frac{dy}{dx} = \cos[\cos\left(\tan\left(\sin^{-1} x\right)\right) \times -\sin$

$= \left(\tan\left(\sin^{-1} x\right)\right) \times \sec^2\left(\sin^{-1} x\right) \times \frac{1}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = -\sec^2\left(\sin^{-1} x\right) \sin\left[\tan\left(\sin^{-1} x\right)\right] \cos\left[\cos\left(\tan\left(\sin^{-1} x\right)\right)\right]$

6. Integrate $\int e^x \cos x dx$

Sol.

Simplify the expression,

Take e^x as the first function and cosx as second function. Then integrating by part, we have

$$I=\int e^x \cos dx = e^x(\sin x)+\int e^x \sin x dx$$
$$= e^x(\sin x)+I_1......(i)$$

Taking e^x and sinx as the first and second functions, respectively, in I_1, we get

$$I_1 = e^x \cos x - \int e^x \cos x dx$$

Substituting the value of I_1 in (i), we get

$$I_1 = e^x \sin x + e^x \cos x - I \quad \text{or } e^x(\sin x+\cos x)$$
$$I=\int e^x \cos x dx = \frac{e^x}{2}(\sin x+\cos x)+C$$
$$I=\frac{e^x}{2}(\sin x+\cos x)+C$$

Hence, the value is $I=\frac{e^x}{2}(\sin x+\cos x)+C$

7. If $\vec{a}=2\vec{i}-3\vec{j}-5\vec{k}$ **and** $\vec{b}=-7\vec{i}+6\vec{j}+8\vec{k}$ **then find** $\vec{a}\times\vec{b}$

Sol.

Simplify the expression,

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & -5\\ -7 & 6 & 8\end{vmatrix}$$
$$\vec{a}\times\vec{b}=(-24+30)\hat{i}+(16-35)\hat{j}+(12-21)\hat{k}$$
$$\vec{a}\times\vec{b}=6\hat{i}-19\hat{j}-9\hat{k}$$

Hence, the expression $\vec{a}\times\vec{b}=6\hat{i}-19\hat{j}-9\hat{k}$.

8. What is the chance of getting 7 or 11 with two dice?

Sol.

Simplify the expression,

A dice has 6 faces, then, two dices has $6\times6=36$ faces So, the sample space on throwing two dice is $n(S)=6\times6=36$

Let A be the event for getting 7, then possible ordered pairs of A are
$A=\{(1,6)(6,1)(2,5)(3,4)(4,3)\}\Rightarrow n(A)=6$

Again, it B be the event for getting then possible ordered pairs of B. are
$B=\{(5,6)(6,5)\}\Rightarrow n(B)=2$

$$\therefore P(A)=\frac{n(A)}{n(S)}=\frac{6}{36}=\frac{1}{6}$$

$$P(B)=\frac{n(B)}{n(S)}=\frac{2}{36}=\frac{1}{18}$$

Probability of getting 7 or 11 is 7 the events A and B are naturally exclusive events, So

$$P(A\cup B)=P(A+B)=P(A)+P(B)=\frac{1}{6}+\frac{1}{18}=\frac{2}{9}$$

Hence, the probability is $\frac{2}{9}$.

Question No. 9 to 12 are of long answer type. Each question carries 7 marks.

Long Answer Type Questions

9. Solve: $\frac{dy}{dx}-\frac{2y}{x}=y^4$ **Or,** Solve $y^2dx+(x^2+xy)dy=0$

Sol.

Now, the given differential equation is of the form $\frac{dx}{dy}+Px=Q$

Where $P=\frac{1}{2y}$ and Q=y^{-4}

$$e^{\int pdy} = e^{\frac{1}{2}\int \frac{dy}{y}} = e^{\log y^{1/2}} = e^{\log \sqrt{y}}$$

$$(Now)\, x \times I.f = \int Q \times I.F + C$$

(Integrating factor) $= \Rightarrow x\sqrt{y} = \int y^{-4} \times \sqrt{y} dy = \int y^{-7/2} dy$

$$\Rightarrow x\sqrt{y} = \frac{y - \frac{7}{2} + 1}{-\frac{7}{2} + 1}$$

$$\Rightarrow x\sqrt{y} = \frac{2}{5} y^{-5/2} + C \Rightarrow x\sqrt{y} = C - \frac{2}{5} y^{-5/2}$$

OR

$$y^2 dx + \left(x^2 + xy\right)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-y^2}{xy + x^2} .....(i)$$

Above equation (i) is a homogeneous differential equation. So point

$y = vx$ then $\frac{dy}{dx} = v + x\frac{dv}{dx}$

This value of dy/dx putting in (i). we get

$$v + x\frac{dv}{dx} = \frac{-v^2 x^2}{vx^2 + x^2} = \frac{-v^2}{v+1}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-v^2}{v+1} - v = \frac{-\left(v^2 - v^2 - v\right)}{v+1} = \frac{v}{v+1}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v}{v+1} = \frac{dx}{x} = \left(\frac{v+1}{v}\right)dv$$

$$\Rightarrow \log x + \log C = \log v = \log ve^x + \log v$$

$$\Rightarrow \log(xC) = \log\left(ve^x\right) \Rightarrow xc + ve^x \Rightarrow xC = \frac{y}{x} e^{y/x}$$

Hence, the value is $xC = \frac{y}{x} e^{y/x}$.

10. Prove that $\int_0^{\pi/2} \log \sin x dx = \int_0^{\pi/2} \log \cos x dx = -\frac{\pi}{2} \log 2$

Sol.

Prove the expression,

Let $I = \int_0^{\pi/2} \log \sin x dx$

Then, by P_4

$$I = \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right)dx = \int_0^{\pi/2} \log \cos x$$

Adding the two value of I, we get

$$2I = \int_0^{\pi/2} (\log \sin x + \log \cos x)dx$$

$$= \int_0^{\pi/2} (\log \sin x \cos x + \log 2 - \log 2)dx \qquad (by \text{ adding or subtracting log2})$$

$$= \int_0^{\pi/2} (\log \sin 2x)dx - \int_0^{\pi/2} (\log 2)dx$$

$put 2x = t$ in the first intergal. then 2 dx=dt, when x=0,t=0 and when x=$\frac{\pi}{2}$ t=π

therefore,

$$2I = \frac{1}{2}\int_0^{\pi/2} (\log \sin t)dt - \frac{\pi}{2}\log 2$$

$$= \frac{1}{2}\int_0^{\pi/2} (\log \sin t)dt - \frac{\pi}{2}\log 2 \qquad [\text{by } P_6 \, as \; \sin(\pi\text{-t})=\text{sint}]$$

$$= \frac{1}{2}\int_0^{\pi/2} (\log \sin x)dx - \frac{\pi}{2}\log 2$$

$$= 1 - \frac{\pi}{2}\log 2$$

$$\int_0^{\pi/2} (\log \sin x)dx = -\frac{\pi}{2}\log 2$$

Hence, prove it.

11. Find the co-ordinates of the point where the line joining the points P(1, -2, 3) and Q(4, 7, 8) cuts the xy-plane.

Sol.

Simplify the expression,

$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + \lambda(4-1)\hat{i} + (7+2)\hat{j} + (8-3)\hat{k} = 0$
$\Rightarrow \vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} + 9\hat{j} + 5\hat{k}\right)$

Let A be the point where the line PQ crosses xy-plane. Then the vector of A is $x\hat{i} + y\hat{j}$ and it must be satisfied the line (1). Now,

$x\hat{i} + y\hat{j} = \hat{i} - 2j + 3k + \lambda\left(3\hat{i} + 9\hat{j} + 5\hat{k}\right)$
$x\hat{i} + y\hat{j} = (1+3\lambda)\hat{i} + (-2+9\lambda)\hat{j} + (3+5\lambda)\hat{k}$

Equating the line coefficient of $\hat{i}, \hat{j}, and\ \hat{k}$ on the both side, we get

$x = (1+3\lambda)$.......(2)
$y = -2+9\lambda$.......(3)
$0 = 3+5\lambda$...........(4)

From equation (4), WE GET $3+4\lambda = 0 \Rightarrow 5\lambda = -3 \Rightarrow \lambda = -\frac{3}{5}$

Putting the value λ in equation (2) and (3), we get

$x = 1+3\lambda = 1-\frac{9}{5} = \frac{5-9}{5} = -\frac{4}{5}$ and
$y = -2+9\lambda = -2-\frac{27}{5} = \frac{-10-27}{5} = \frac{-37}{5}$

Thus, the straight line the joining the points P(1, -2, 3) and Q (4, 7, 8)

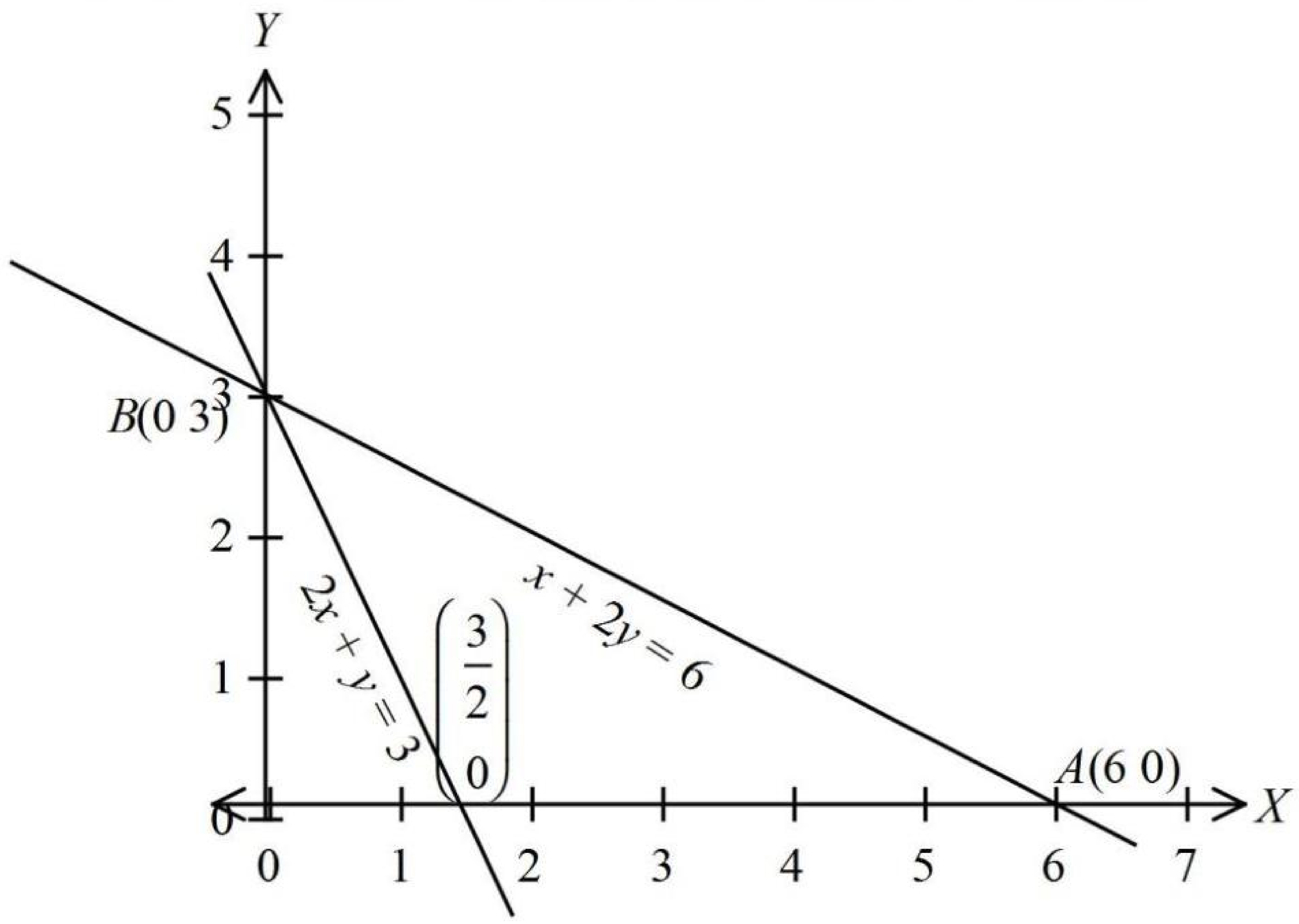
Crosses xy-plane at the point $\left(-\frac{4}{5}, \frac{-37}{5}, 0\right)$.

12. Minimize : $Z = x + 2y$

Subject to

$2x + y \geq 3$
$x + 2y \geq 6$
$x, y \geq 0$

Sol.



Simplify the expression,

We have $Z = x + 2y$

Subject to constrains

$2x + y \geq 3 \Rightarrow 2x + y = 3$........(1)
$x + 2y \geq 6 \Rightarrow x + 2y = 6$.........(2)
$x, y \geq 0 \Rightarrow x = 0, y = 0$............(3)

First of all draw the graph of linear equation corresponds to linear inequation as shown in figure.

It is clear from figure the line $2x + y = 3$ passes through the points (3/2,0) and (0,3)

On putting x=0, y=0 in $2x + y \geq$, we get $0 \geq 3$ which is not true.

The region $2x + y \geq$ lies on and above the line.

Similarly the line $x + 2y = 6$ passes through the points A(6,0) and B(0,3)

On putting x=0, y=0 in $x + 2y \geq 6$ we get $0 \geq 6$ which is not true.

$x + 2y \geq 6$ lies on and above the line lie $x \geq 0$ s on and right of y-axis $y \geq 0$ lies on and above the x-axis So, shaded region X ABY above the line AB is feasible region Now, from the points A(6,0) and B(0,3) to find the minimum value of $z = x + 2y$ as below.

$Z = x + 2y$

Points $A(6,0)$ Z=6+2.0=6

$B(0,3)$ Z=0+2.3=6

sHence, the common minimum value of the objective function Z is 6.