PAPER-2 B. Arch. (Mathematics & Aptitute Test)

- **Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.**
- **You are not allowed to leave the Examination Hall before the end of the test.**

Important Instructions :

1. Immediately fill in the particulars on this page of the Test Booklet with *Blue/Black Ball Point Pen.*
2. This Test Booklet consists of three parts - **Part I, Part II** and **Part III. Part I** has **30** objective type questions of Mathematics consisting of **FOUR (4)** marks for each correct response. **Part II** Aptitude Test has **50** objective type questions consisting of **FOUR (4)** marks for each correct response. Mark your answers for these questions in the appropriate space against the number corresponding to the question in the Answer Sheet placed inside this Test Booklet. ***Use Blue/ Black Ball Point Pen only for writing particulars/marking responses on Side-1 and Side-2 of the Answer Sheet.*** **Part III** consists of **2** questions carrying **70** marks which are to be attempted on a separate Drawing Sheet which is also placed inside the Test Booklet. Marks allotted to each question are written against each question. ***Use colour pencils or crayons only on the Drawing Sheet. Do not use water colours.*** For each ***incorrect response*** in Part I and Part II, ***one-fourth*** **(¼)** of the total marks allotted to the question would be deducted from the total score. ***No deduction*** from the total score, however, will be made ***if no response*** is indicated for an item in the Answer Sheet.
3. There is only one correct response for each question in Part I and Part II. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 2 above.
4. The test is of **3 hours** duration. The maximum marks are **390.**
5. On completion of the test, the candidates must hand over the Answer Sheet of **Mathematics** and **Aptitude Test-Part I & II** and the **Drawing Sheet** of **Aptitude Test-Part III** alongwith **Test Booklet** for **Part III** to the Invigilator in the Room/Hall. Candidates are allowed to take away with them the Test Booklet of **Aptitude Test-Part I & II.**
6. The CODE for this Booklet is **K**. Make sure that the CODE printed on **Side-2** of the Answer Sheet and on the Drawing Sheet **(Part III)** is the same as that on this booklet. Also tally the Serial Number of the Test Booklet, Answer Sheet and Drawing Sheet and ensure that they are same. In case of discrepancy in Code or Serial Number, the candidate should immediately report the matter to the Invigilator for replacement of the Test Booklet, Answer Sheet and the Drawing Sheet.

महत्त्वपूर्ण निर्देश :

1. परीक्षा पुस्तिका के इस पृष्ठ पर आवश्यक विवरण *नीले/काले बॉल पाइंट पेन* से तत्काल भरें।
2. इस परीक्षा पुस्तिका के तीन भाग हैं – **भाग I, भाग II** एवं **भाग III.** पुस्तिका के **भाग I** में गणित के **30** वस्तुनिष्ठ प्रश्न हैं जिसमें प्रत्येक प्रश्न के सही उत्तर के लिये **चार (4)** अंक निर्धारित किये गये हैं। **भाग II** अभिरुचि परीक्षण में **50** वस्तुनिष्ठ प्रश्न हैं जिनमें प्रत्येक सही उत्तर के लिए **चार (4)** अंक हैं। इन प्रश्नों का उत्तर इस परीक्षा पुस्तिका में रखे उत्तर पत्र में संगत क्रम संख्या के गोले में गहरा निशान लगाकर दीजिए। ***उत्तर पत्र के पृष्ठ-1 एवं पृष्ठ-2 पर वांछित विवरण लिखने एवं उत्तर अंकित करने हेतु केवल नीले/ काले बॉल पाइंट पेन का ही प्रयोग करें।*** पुस्तिका के **भाग III** में **2** प्रश्न हैं जिनके लिए **70** अंक निर्धारित हैं। यह प्रश्न इसी परीक्षा पुस्तिका के अन्दर रखी ड्राइंग शीट पर करने हैं। प्रत्येक प्रश्न हेतु निर्धारित अंक प्रश्न के सम्मुख अंकित हैं। ***ड्राइंग शीट पर केवल रंगीन पेंसिल अथवा क्रेयोन का ही प्रयोग करें। पानी के रंगों का प्रयोग न करें।*** भाग I और भाग II में प्रत्येक ***गलत उत्तर*** के लिए उस प्रश्न के लिए निर्धारित कुल अंकों में से ***एक-चौथाई*** **(¼)** अंक कुल योग में से काट लिए जाएँगे। यदि उत्तर पत्र में किसी प्रश्न का कोई ***उत्तर नहीं*** दिया गया है, तो कुल योग में से कोई ***अंक नहीं काटे*** जाएँगे।
3. इस परीक्षा पुस्तिका के भाग I और भाग II में प्रत्येक प्रश्न का केवल एक ही सही उत्तर है। एक से अधिक उत्तर देने पर उसे गलत उत्तर माना जायेगा और उपरोक्त निर्देश 2 के अनुसार अंक काट लिये जायेंगे।
4. परीक्षा की अवधि **3 घंटे** है। अधिकतम अंक **390** हैं।
5. परीक्षा समाप्त होने पर, परीक्षार्थी **गणित** एवं **अभिरुचि परीक्षण-भाग I** एवं **II** का उत्तर पत्र एवं **अभिरुचि परीक्षण-भाग III** की **ड्राइंग शीट** एवं **परीक्षा पुस्तिका भाग III** हाल/कक्ष निरीक्षक को सौंपकर ही परीक्षा हाल/कक्ष छोड़ें। परीक्षार्थी **अभिरुचि परीक्षण-भाग I** एवं **II** की परीक्षा पुस्तिका अपने साथ ले जा सकते हैं।
6. इस पुस्तिका का **संकेत K** है। यह सुनिश्चित कर लें कि इस पुस्तिका का **संकेत**, उत्तर पत्र के **पृष्ठ-2** एवं ड्राइंग शीट **(भाग-III)** पर छपे संकेत से मिलता है। यह भी सुनिश्चित कर लें कि परीक्षा पुस्तिका, उत्तर पत्र एवं ड्राइंग शीट पर क्रम संख्या मिलती है। अगर संकेत या क्रम संख्या भिन्न हों, तो परीक्षार्थियों को निरीक्षक से दूसरी परीक्षा पुस्तिका, उत्तर पत्र एवं ड्राइंग शीट लेने के लिए उन्हें तुरन्त इस त्रुटि से अवगत कराएँ।

Name of the Candidate (in Capitals) : _____
परीक्षार्थी का नाम (बड़े अक्षरों में) :

Roll Number : in figures
अनुक्रमांक : अंकों में | | | | | | | | |

: in words _____
: शब्दों में

Examination Centre Number : | | | | | | |
परीक्षा केन्द्र नम्बर :

Centre of Examination (in Capitals) : _____
परीक्षा केन्द्र (बड़े अक्षरों में) :

Candidate's Signature : _____
परीक्षार्थी के हस्ताक्षर :

Invigilator's Signature (1) : _____
निरीक्षक के हस्ताक्षर (1) :

Invigilator's Signature (2) : _____
निरीक्षक के हस्ताक्षर (2) :

PART-I
(MATHEMATICS)

1. The statement $(\sim p \Rightarrow q) \Rightarrow (\sim p \wedge q)$ is a
(a) contradiction (b) tautology
(c) Neither (a) nor (b) (d) None of these

2. If $\int x \log\left(1+\frac{1}{x}\right)dx$
$= f(x)\log(x+1) + g(x)x^2 + xL + c$, then
(a) $L=1$ (b) $f(x)=\frac{x^2}{2}$
(c) $g(x)=\log x$ (d) No ne of these

3. The value of integral $\left|\int_0^{\pi}[2\sin x]\,dx\right|$, where [x] denotes greatest integer function, is
(a) $\frac{2\pi}{3}$ (b) 2π (c) $\frac{\pi}{2}$ (d) None of these

4. Area bounded by the loop of the curve $y^2 = x^2(1-x)$, is
(a) $\frac{16}{15}$ sq units (b) $\frac{8}{15}$ sq unit
(c) $\frac{3}{7}$ sq unit (d) $\frac{7}{15}$ sq unit

5. The solution of differential equation
$2y\sin x\left(\frac{dy}{dx}\right) = 2\sin x\cos x - y^2\cos x$ at $x=\frac{\pi}{2}$, $y=1$ is
(a) $y^2=\sin x$ (b) $y=\sin^2 x$
(c) $y^2=\cos x+1$ (d) None of these

6. Let $x_1, x_2, \ldots, x_n$ be n observations such that $\Sigma x_i^2 = 400$ and $\Sigma x_i = 80$. Then a possible value of n among the following, is
(a) 12 (b) 14 (c) 16 (d) 18

7. An experiment succeeds twice as often as it fails. Then the probability that in the, next 4 trials there will be atleast 2 successes, is
(a) $\frac{1}{9}$ (b) $\frac{8}{9}$ (c) $\frac{5}{9}$ (d) $\frac{2}{9}$

8. The value of $\sin(3\sin^{-1}(0.8))$ is
(a) sin (2) (b) sin (1.88)
(c) $-\sin(0.88)$ (d) None of these

9. A vertical lamp post of height 9 m stands at the corner of a rectangular field. The angle of elevation of its top from the farthest corner is 30°, whereas from another corner it is 45°. The area of rectangular field is
(a) $81\sqrt{2}$ m^2 (b) $80\sqrt{2}$ m^2
(c) 81 m^2 (d) $50\sqrt{3}$ m^2

10. Let **a**, **b** and **c** are three mutually perpendicular unit vectors and **d** is a unit vector which makes equal angle with **a**, **b** and **c**, then $|\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}|^2$ is equal to
(a) 4 (b) $4\pm\sqrt{3}$
(c) $4\pm 2\sqrt{3}$ (d) None of these

11. If a variable plane forms a tetrahedron of constant value $64k^3$ with the coordinate planes, then the locus of the centroid of the tetrahedron, is
(a) $xyz=k^3$ (b) $xyz=2k^3$
(c) $xyz=12k^3$ (d) $xyz=6k^3$

12. The equation of the tangent to the curve $4x^2-9y^2=1$ which is parallel to $4y=5x+7$ is
(a) $24y-30x=17$ (b) $30y-24x=\pm\sqrt{61}$
(c) $24y-30x=\pm\sqrt{161}$ (d) None of the above

13. The equation of the locus of the pole with respect to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, of any tangent line to the auxiliary circle is the curve $\frac{x^2}{a^4}+\frac{y^2}{b^2}=\lambda^2$, where
(a) $\lambda^2=a^2$ (b) $\lambda^2=\frac{1}{a^2}$ (c) $\lambda^2=b^2$ (d) $\lambda^2=\frac{1}{b^2}$

14. The solution set for $\frac{(2x+3)(4-3x)^3(x-4)}{(x-2)^2x^5}\le 0$
(a) $\left(-\infty,\frac{-3}{2}\right)\cup\left(0,\frac{4}{3}\right)\cup(4,\infty)$
(b) $\left(-\frac{3}{2},0\right)\cup\left(\frac{4}{3},4\right)$
(c) $(-\infty,0)\cup(2,\infty)$
(d) None of the above

15. Any point on the parabola whose focus is (0, 1) and the directrix is $x+2=0$ is given by
(a) $(t^2+1, 2t-1)$ (b) $(t^2+1, 2t+1)$
(c) $(t^2, 2t)$ (d) None of these

16. The equation of the image of the circle $x^2+y^2+16x-24y+183=0$ by the line mirror $4x+7y+13=0$, is
(a) $x^2+y^2+32x-4y-235=0$
(b) $x^2+y^2+32x+4y+235=0$
(c) $x^2+y^2+32x-4y+235=0$
(d) None of the above

17. The coordinates of the points O, A and B are (0, 0), (0, 4) and (6, 0) respectively. If a point P moves in such a way that area of ΔPOA is double of area of ΔPOB, then the locus of point P is
(a) $(3x-y)(3x+y)=0$ (b) $(x-3y)(x+y)=0$
(c) $(x-3y)(x+3y)=0$ (d) $(3x-y)(x+y)=0$

18. If z_1, z_2 and z_3 are the roots of the equation $x^3+3ax^2+3bx+c=0$ in which a, b and c are complex numbers, correspond to the points A, B and C on gaussian plane. Then, the ΔABC is equilateral, if
(a) $a^2=b$ (b) $a=b$
(c) $a=b^2$ (d) None of These

19. If $\sqrt[x]{a} = \sqrt[y]{b} = \sqrt[z]{c}$ and a, b and c are in GP, then x, y and z will be in
(a) AP (b) GP
(c) HP (d) AGP

20. Four boys picked up 30 mangoes. In how many ways can they divide them, if all mangoes be identical?
(a) 5450 (b) 5453 (c) 5455 (d) 5456

21. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, then $C_0^2 + \frac{C_1^2}{2} + \frac{C_2^2}{3} + \ldots + \frac{C_n^2}{n+1}$ is equal to
(a) $\frac{(2n)!}{\{(n+1)!\}^2}$ (b) $\frac{(2n+1)!}{\{(n+1)!\}^2}$
(c) $\frac{(2n-1)!}{\{(n+1)!\}^2}$ (d) None of these

22. The value of $\begin{vmatrix} yz & zx & xy \\ p & 2q & 3r \\ 1 & 1 & 1 \end{vmatrix}$, where x, y and z are respectively pth, $(2q)$th and $(3r)$th terms of an HP, is
(a) 0 (b) -1
(c) 1 (d) None of these

23. If $f(x+y) = f(x)\,f(y)$ for all $x, y \in R$, $f(5) = 2$, $f'(0) = 3$. Then, $f(5)$ is equal to
(a) 3 (b) 5 (c) 6 (d) 8

24. If $y = \cos^{-1}(\cos x)$, then $\frac{dy}{dx}$ at $x = \frac{5\pi}{4}$ is equal to
(a) 1 (b) -1 (c) $\frac{1}{\sqrt{2}}$ (d) None of these

25. Let $f(x) = \begin{cases} |x|, \text{ for } 0 < |x| \le 2 \\ 1, \text{ for } x = 0 \end{cases}$, then at $x = 0$, f has
(a) local maximum (b) local minimum
(c) no local minimum (d) no extreme

Directions (Q. Nos. 26 to 30) *These questions are Assertion-Reason type questions. Each of these questions contains two statements, Statement* I *(Assertion) and Statement* II *(Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.*

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.
(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

26. **Statement I** The number of solutions of $\sin x = x^2 + x + 1$ is 0.

Statement II While finding a solution, avoid square and denominator function.

27. **Statement I** If a and b are positive integers and $[x]$ denotes greatest integer $\le x$, then $\lim_{x \to 0^+} \frac{x}{a}\left[\frac{b}{x}\right] = \frac{b}{a}$.

Statement II $\lim_{x \to \infty} \frac{\{x\}}{x} \to 0$ where $\{x\}$ denotes fractional part of x.

28. **Statement I** Matrix $[A]_{3\times3}$, is defined as $a_{ij} = \frac{i-j}{i+2j}$, which cannot be expressed as a sum of symmetric and skew-symmetric matrix.

Statement II Matrix $[A]_{3\times3}$, $a_{ij} = \frac{i-j}{i+2j}$ is neither symmetric nor skew-symmetric. matric.

29. **Statement I** $\lim_{x \to 0}\left[\frac{\tan^{-1}x}{x}\right] = 0$ where $[\,]$ represents greatest integer function.

Statement II $\frac{\tan^{-1}x}{x} < 1$ in the neighbourhood of $x = 0$

30. Suppose $f(x) = x\sin x - \frac{1}{2}\sin^2 x$, $x \in \left(0, \frac{\pi}{2}\right)$

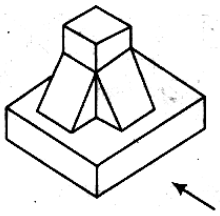
Statement I Range of $f(x)$ is $0, \frac{\pi-1}{2}$.

Statement II Range of $f(x)$ is not determined.

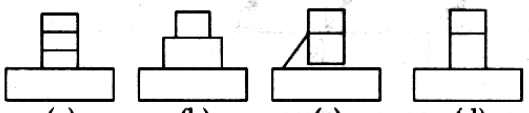
PART-II
(APTITUDE TEST)

Directions (Q. Nos. 31 to 33) *The 3-D problem figure shows a view of an object. Identify the correct front view, from amongst the answer figures, looking the direction of arrow.*

31. **Problem Figure**

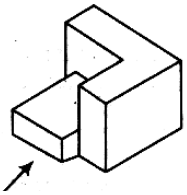


Answer Figures

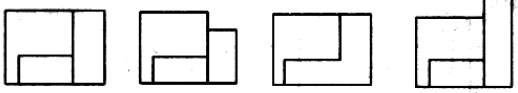


(a) (b) (c) (d)

32. **Problem Figure**

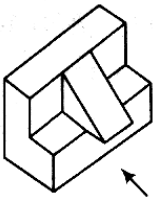


Answer Figures

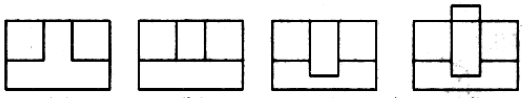


(a) (b) (c) (d)

33. Problem Figure



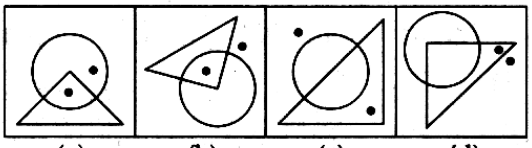
Answer Figures



(a) (b) (c) (d)

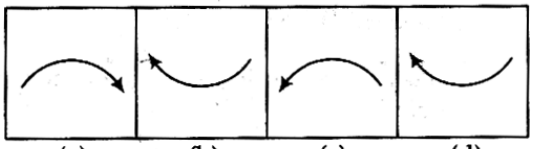
Directions (Q. Nos. 34 to 36) *Find the odd figure out of the problem figure given below.*

34.



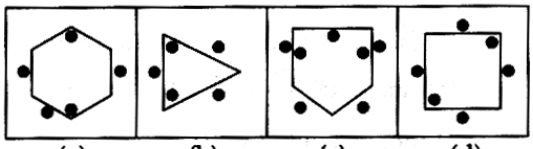
(a) (b) (c) (d)

35.



(a) (b) (c) (d)

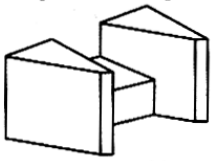
36.



(a) (b) (c) (d)

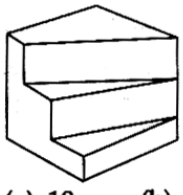
Directions (Q. Nos. 37 to 41) *Find out the total number of surfaces of the object given below in the problem figure.*

37.



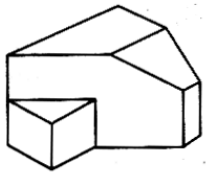
(a) 15 (b) 12 (c) 17 (d) 19

38.



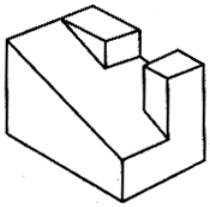
(a) 10 (b) 10 (c) 13 (d) 14

39.



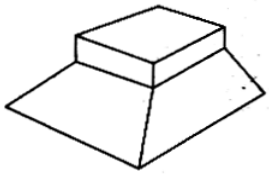
(a) 9 (b) 10 (c) 12 (d) 11

40.



(a) 15 (b) 16 (c) 12 (d) 14

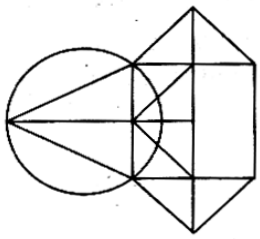
41.



(a) 8 (b) 9 (c) 10 (d) 11

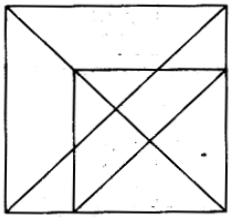
Directions (Q. Nos. 42 and 43) *How many total number of triangle are there in the problem figure given below?*

42.



(a) 13 (b) 16 (c) 15 (d) 14

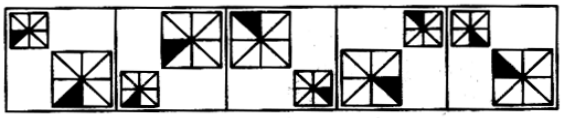
43.



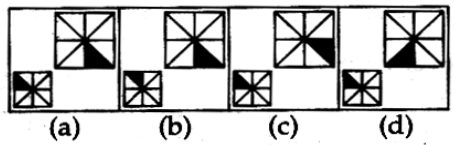
(a) 21 (b) 22 (c) 19 (d) 20

Directions (Q. Nos. 44 to 47) *Which one of the answer figures will complete the sequence of the problem figures?*

44. Problem Figures

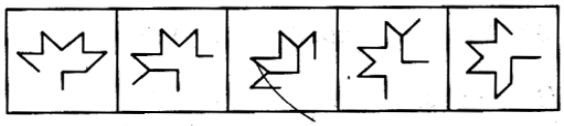


Answer Figures

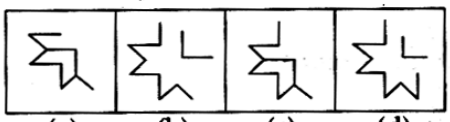


(a) (b) (c) (d)

45. Problem Figures

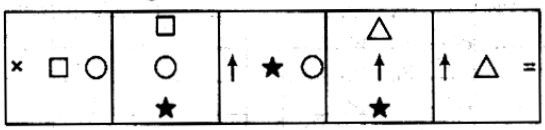


Answer Figures

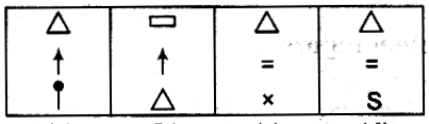


(a) (b) (c) (d)

46. Problem Figures

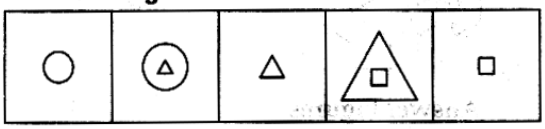


Answer Figures

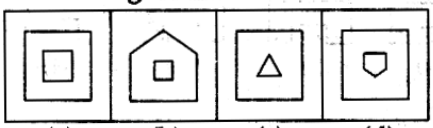


(a) (b) (c) (d)

47. Problem Figures



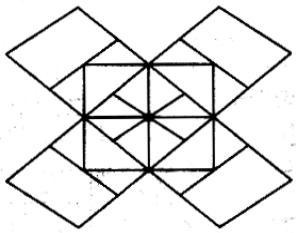
Answer Figures



(a) (b) (c) (d)

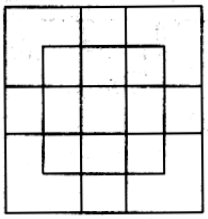
Directions (Q. Nos. 48 and 49) *How many total number of squares are there in the problem figure given below?*

48.



(a) 26 (b) 22 (c) 17 (d) 18

49.

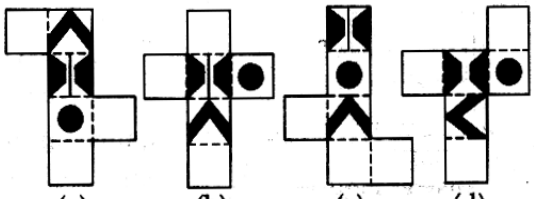


(a) 27 (b) 28 (c) 30 (d) 21

Directions (Q. Nos. 50 to 52) *Which one of the answer figures, shows the correct view of the 3-D problem figure, after the problem figure open up?*

50. Problem Figure

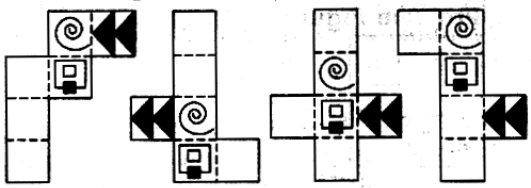
Answer Figures



(a) (b) (c) (d)

51. Problem Figure

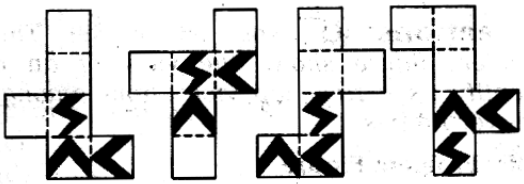
Answer Figures



(a) (b) (c) (d)

52. Problem Figure

Answer Figures

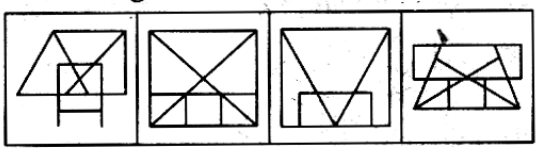


(a) (b) (c) (d)

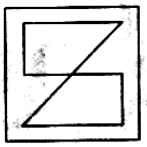
Directions (Q. Nos. 53 to 55) *Following questions figure is hidden in one of the answer answer figures, in the same size and direction. Select which one is correct.*

53. Problem Figure

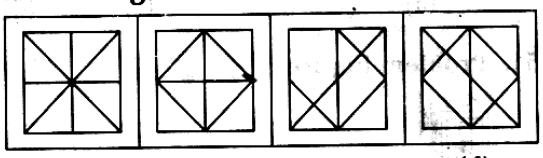
Answer Figures



54. Problem Figure



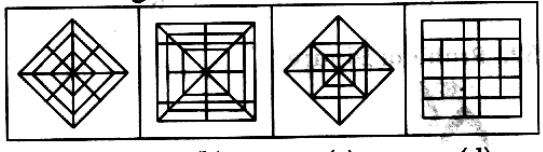
Answer Figures



(a) (b) (c) (d)

55. Problem Figure

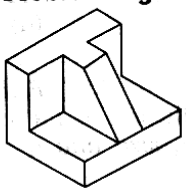
Answer Figures



(a) (b) (c) (d)

Directions (Q. Nos. 56 to 58) *The 3-D problem figure shows the view of an object. Identify its correct top view, from amongst the answer figures.*

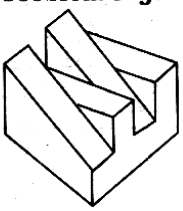
56. Problem Figure



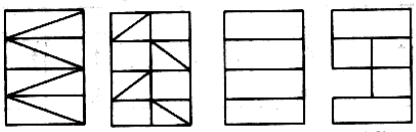
Answer Figures

(a) (b) (c) (d)

57. Problem Figure

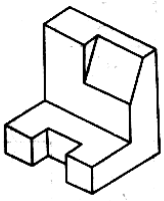


Answer Figures



(a) (b) (c) (d)

58. Problem Figure

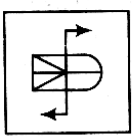


Answer Figures

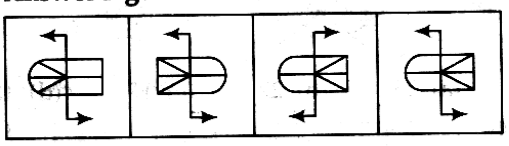
(a) (b) (c) (d)

Directions (Q. Nos. 59 to 62) *Which one of answer figures is the correct mirror image of the problem figure?*

59. Problem Figure

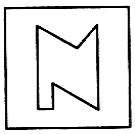


Answer Figures

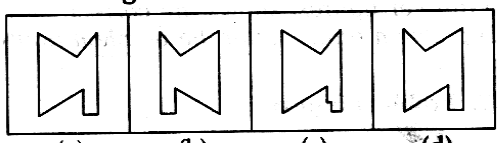


(a) (b) (c) (d)

60. Problem Figure

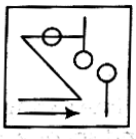


Answer Figures

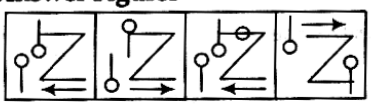


(a) (b) (c) (d)

61. Problem Figure

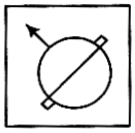


Answer Figures

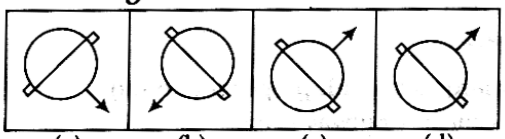


(a) (b) (c) (d)

62. Problem Figure



Answer Figures



(a) (b) (c) (d)

63. Rath Yatra is a major festival in the temple at
(a) Dwarka (b) Ayodhya
(c) Puri (d) Varanasi

64. The World Telecommunication Day is observed on
(a) May 19 (b) May 17
(c) May 15 (d) May 13

65. Ranbaxy Company is being asquired by a company of
(a) India (b) Germany
(c) France (d) Japan

66. Blindness is caused by the deficiency of
(a) Vitamin C (b) Vitamin A
(c) Vitamin D (d) Vitamin B

67. The term 'Butterfly Stroke' is referred to which game?
(a) Tennis (b) Table Tennis
(c) Swimming (d) Football

68. The first Kanshi Ram Khel Ratna Award was given to
(a) Praveen Kumar (b) Sachin Tendulkar
(c) MS Dhoni (d) Yuvraj Singh

69. In the 15th Asian Games India bagged the largest number of medals in
(a) Shooting (b) boxing
(c) Swimming (d) Athletics

70. Which country's government report is knwon as yellow book?
(a) Germany (b) France
(c) Italy (d) Russia

71. "Architecture is the art of how to waste space." This is a quote from what no-nonsense modernist?
(a) Le Corbusier (b) Alvar Aalto
(c) Jacobus Oud (d) Philip Johnson

72. "Whenever I draw a circle, I immediately want to step out of it." This statement is rather ironic, considering the source ...
(a) R. Buckminster Fuller
(b) Bertrand Goldberg
(c) Frank Gehry
(d) Fank Lloyd Wright

73. 'Cantilever', 'suspension' and 'swing' are all types of what common architectural construct?
(a) Bridge (b) Stadium
(c) Skyscrapers (d) Monument

74. If you would like to become an architect, you will be required to take classes in which of these disciplines?
(a) Masonry (b) Metalworking
(c) Electronics (d) Drafting

75. Which architect designed the Kimball Art Museum (Fort Worth, Texas)?
(a) Walter Gropius
(b) Frank Lloyd Wright
(c) Louis Kahn
(d) William Chambers

76. Which architect designed the Kaufmann Desert House (Palm Springs, California)?
(a) Frank Lloyd Wright
(b) Louis Kahn
(c) Kewisgwi Natsugi
(d) Richard Neutra

77. Char Minar is located at
(a) Hyderabad (b) Aurangabad
(c) Allahabad (d) New Delhi

78. Where is 'Hawa Mahal' located?
(a) Jaipur (b) Udaipur
(c) Bijapur (d) Delhi

79. Sydney Opera Home was designed by
(a) Jon Utzon (b) Eiro Saarinen
(c) Walter Gropius (d) FL Wright

80. The World Trade Center in New York, USA was designed by
(a) Minoru Yamasaki (b) PL Nevi
(c) Richard Neutra (d) Confucius

PART-III
(Drawing Test)

1. (a) Five bricks, four circular wooden loge and six balls are given to you. Make an interesting three dimensional stable composition using these elements and show the effect of light and shadow on the composition.
(b) Draw a mirror image of the sketch on the other side of the centre line.

2. Sketch from memory the following to a sufficiently large size with proper rendering.
(a) Scene market (b) Scene of a railway station.

Read the following instructions carefully :	*निम्नलिखित निर्देश ध्यान से पढ़ें :*
1. **Part I** has **30** objective type questions of Mathematics consisting of **FOUR (4)** marks each for each correct response. **Part II** (Aptitude Test) has **50** objective type questions consisting of **FOUR (4)** marks for each correct response. **Part III** consists of 2 questions carrying **70** marks which are to be attempted on a separate Drawing Sheet which is also placed inside this Test Booklet. Marks allotted to each question are written against each question. For each ***incorrect response*** in Part I and Part II, ***one-fourth*** (¼) of the total marks allotted to the question would be deducted from the total score. ***No deduction*** from the total score, however, will be made ***if no response*** is indicated for an item in the Answer Sheet.	1. पुस्तिका के **भाग I** में गणित के **30** वस्तुनिष्ठ प्रश्न हैं जिसमें प्रत्येक प्रश्न के सही उत्तर के लिए **चार (4)** अंक निर्धारित किये गये हैं। भाग II (अभिरुचि परीक्षण) में **50** वस्तुनिष्ठ प्रश्न हैं जिनमें प्रत्येक सही उत्तर के लिए **चार (4)** अंक हैं। पुस्तिका के **भाग III** में **2** प्रश्न हैं जिनके लिए **70** अंक निर्धारित हैं। यह प्रश्न इसी परीक्षा पुस्तिका के अन्दर रखी ड्राइंग शीट पर करने हैं। प्रत्येक प्रश्न हेतु निर्धारित अंक प्रश्न के सम्मुख अंकित हैं। भाग I और भाग II में प्रत्येक ***गलत उत्तर*** के लिए उस प्रश्न के लिए निर्धारित कुल अंकों में से ***एक-चौथाई*** (¼) अंक कुल योग में से काट लिए जाएँगे। यदि उत्तर पत्र में किसी प्रश्न का कोई ***उत्तर नहीं*** दिया गया है, तो कुल योग में से कोई ***अंक नहीं काटे*** जाएँगे।
2. Handle the Test Booklet, Answer Sheet and Drawing Sheet with care, as under no circumstances (except for discrepancy in Test Booklet Code and Answer Sheet Code), another set will be provided.	2. परीक्षा पुस्तिका, उत्तर पत्र एवं ड्राइंग शीट का ध्यानपूर्वक प्रयोग करें, क्योंकि किसी भी परिस्थिति में (केवल परीक्षा पुस्तिका एवं उत्तर पत्र के कोड में भिन्नता की स्थिति को छोड़कर) दूसरी परीक्षा पुस्तिका उपलब्ध नहीं करायी जाएगी।
3. The candidates are not allowed to do any rough work or writing work on the Answer Sheet. All calculations/ writing work are to be done on the space provided for this purpose in the Test Booklet itself, marked 'Space for Rough Work'. This space is given at the bottom of each page and in **3** pages (pages **25 - 27**)at the end of the booklet.	3. परीक्षार्थियों को उत्तर पत्र पर कोई भी रफ कार्य या लिखाई का काम करने की अनुमति नहीं है। सभी गणना एवं लिखाई का काम, परीक्षा पुस्तिका में निर्धारित जगह जो कि 'रफ कार्य के लिए जगह' द्वारा नामांकित है, पर ही किया जायेगा। यह जगह प्रत्येक पृष्ठ पर नीचे की ओर तथा पुस्तिका के अंत में **3** पृष्ठों (पृष्ठ **25 - 27**) पर दी गई है।
4. Each candidate must show on demand his/her Admit Card to the Invigilator.	4. पूछे जाने पर प्रत्येक परीक्षार्थी निरीक्षक को अपना प्रवेश कार्ड दिखाएँ।
5. No candidate, without special permission of the Superintendent or Invigilator, should leave his/her seat.	5. अधीक्षक या निरीक्षक की विशेष अनुमति के बिना कोई परीक्षार्थी अपना स्थान न छोड़ें।
6. On completion of the test, the candidates should not leave the examination hall without handing over their Answer Sheet of Mathematics and Aptitude Test-Part I & II and Drawing Sheet of Aptitude Test-Part III to the Invigilator on duty and sign the Attendance Sheet at the time of handing over the same. Cases where a candidate has not signed the Attendance Sheet the second time will be deemed not have handed over these documents and dealt with as an unfair means case. **The candidates are also required to put their left hand THUMB impression in the space provided in the Attendance Sheet.** However, the candidates are allowed to take away with them the Test Booklet of Mathematics and Aptitude Test - Part I & II.	6. परीक्षा समाप्त होने पर, परीक्षार्थी निरीक्षकों को अपने गणित - भाग I एवं अभिरुचि परीक्षण - भाग II का उत्तर पत्र एवं अभिरुचि परीक्षण-भाग III की ड्राइंग शीट देने और उपस्थिति पत्र पर अपने हस्ताक्षर दोबारा करने के पश्चात् ही परीक्षा हाल छोड़ें। ऐसा न करने पर यह माना जायेगा कि उत्तर पत्र एवं ड्राइंग शीट नहीं लौटाए गए हैं जिसे अनुचित साधन प्रयोग की श्रेणी में माना जायगा। **परीक्षार्थी अपने बायें हाथ के अंगूठे का निशान उपस्थिति पत्र में दिए गए स्थान पर अवश्य लगाएँ।** तथापि, परीक्षार्थी अपनी गणित एवं अभिरुचि परीक्षण - भाग I एवं II की परीक्षा पुस्तिका को ले जा सकते हैं।
7. Use of Electronic/Manual Calculator or drawing instruments (such as scale, compass etc.) is not allowed.	7. इलेक्ट्रॉनिक/हस्तचालित परिकलक या ड्राइंग उपकरण (जैसे कि स्केल, कंपास इत्यादि) का प्रयोग वर्जित है।
8. The candidates are governed by all Rules and Regulations of the JAB/Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the JAB/Board.	8. परीक्षा हाल में आचरण के लिए परीक्षार्थी ज.ए.ब./बोर्ड के नियमों एवं विनियमों द्वारा नियमित होंगे। अनुचित साधन प्रयोग के सभी मामलों का फैसला ज.ए.ब./बोर्ड के नियमों एवं विनियमों के अनुसार होगा।
9. No part of the Test Booklet, Answer Sheet and Drawing Sheet shall be detached/folded or defaced under any circumstances.	9. किसी भी स्थिति में परीक्षा पुस्तिका, उत्तर पत्र एवं ड्राइंग शीट का कोई भी भाग न तो अलग किया जाएगा और न ही मोड़ा जायेगा अथवा बिगाड़ा जायेगा।
10. The candidates will write the Test Booklet Number as given in the Test Booklet, Answer Sheet and Drawing Sheet in the Attendance Sheet also.	10. परीक्षा पुस्तिका, उत्तर पत्र एवं ड्राइंग शीट में दी गई परीक्षा पुस्तिका संख्या को परीक्षार्थी सही तरीके से हाज़िरी पत्र में भी लिखें।
11. **Candidates are not allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, electronic device or any other material except the Admit Card inside the examination hall/room.**	11. **परीक्षार्थी द्वारा परीक्षा हॉल/कक्ष में प्रवेश कार्ड के सिवाय किसी प्रकार की पाठ्य सामग्री, मुद्रित या हस्तलिखित, कागज की पर्चियाँ, पेजर, मोबाइल फोन, इलेक्ट्रॉनिक उपकरण या किसी अन्य प्रकार की सामग्री को ले जाने या उपयोग करने की अनुमति नहीं है।**

SOLUTIONS

1. (c)

p	q	$\sim p$	$\sim p \Rightarrow q$	$\sim p \wedge q$	$(\sim p \Rightarrow q) \Leftrightarrow (\sim p \wedge q)$
T	T	F	T	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	F	F	F

2. (d) Let $I = \int \underset{II}{x} \log \underset{I}{\left(1+\frac{1}{x}\right)} dx$

$$= \log\left(1+\frac{1}{x}\right)\frac{x^2}{2} - \int \frac{x^2}{2}\cdot\frac{1}{\left(1+\frac{1}{x}\right)}\left(-\frac{1}{x^2}\right)dx$$

$$= \frac{x^2}{2}\log\left(1+\frac{1}{x}\right) + \frac{1}{2}\int \frac{x}{(1+x)}dx$$

$$= \frac{x^2}{2}\log\left(1+\frac{1}{x}\right) + \frac{1}{2}\left[\int dx - \int \frac{1}{1+x}dx\right]$$

$$= \frac{x^2}{2}\log\left(\frac{x+1}{x}\right) + \frac{1}{2}x - \log(1+x) + C$$

$$= \log(1+x)\left(\frac{x^2}{2}-1\right) - \frac{x^2}{2}\log x + \frac{x}{2} + C$$

$\therefore \quad f(x) = \left(\frac{x^2}{2}-1\right),\ g(x) = -\frac{\log x}{2}$

$$L = \frac{1}{2}$$

3. (a) Let $I = \int_0^{\pi}[2\sin x]\,dx$

$$= \int_0^{\pi/6}[2\sin x]\,dx + \int_{\pi/6}^{\pi/2}[2\sin x]\,dx + \int_{\pi/2}^{5\pi/6}[2\sin x]\,dx + \int_{5\pi/6}^{\pi}[2\sin x]\,dx$$

$$= 0 + \int_{\pi/6}^{\pi/2} 1\,dx + \int_{\pi/2}^{5\pi/6} 1\,dx + 0$$

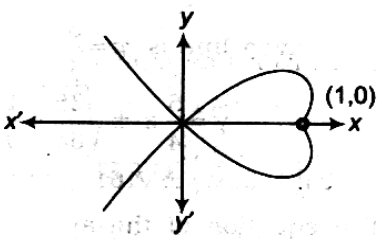
$$= [x]_{\pi/6}^{\pi/2} + [x]_{\pi/2}^{5\pi/6}$$

$$= \frac{\pi}{2} - \frac{\pi}{6} + \frac{5\pi}{6} - \frac{\pi}{2} = \frac{2\pi}{3}$$

4. (b) Given curve is $y^2 = x^2(1-x)$. Here, y is even so curve is symmetrical about the x-axis.

$\therefore$ Area $= 2\int_0^1 y\,dx$

$$= 2\int_0^a x\sqrt{1-x}\,dx$$



Put $\quad x = \sin^2\theta \Rightarrow dx = 2\sin\theta\cos\theta\,d\theta$

$\therefore \quad A = 2\int_0^{\pi/2} \sin^2\theta\sqrt{1-\sin^2\theta}\,2\sin\theta\cos\theta\,d\theta$

$$= 4\int_0^{\pi/2}\sin^3\theta\cos^2\theta\,d\theta$$

$$= 4\left[\frac{\Gamma\left(\frac{3+1}{2}\right)\Gamma\left(\frac{2+1}{2}\right)}{2\Gamma\left(\frac{3+2+2}{2}\right)}\right]$$

$$= 4\left[\frac{1\cdot\frac{1}{2}\cdot\sqrt{\pi}}{2\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}}\right]$$

$$= \frac{8}{15}\text{ sq unit}$$

5. (a) Given equation can be rewritten as

$$2y\frac{dy}{dx} + y^2\cot x = \cos x$$

Put $\quad y^2 = v \Rightarrow 2y\frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \quad \frac{dv}{dx} + v\cot x = \cos x$

$\therefore \quad \text{IF} = e^{\int\frac{\cos x}{\sin x}dx} = e^{\log\sin x} = \sin x$

$\therefore$ Solution is

$$v\sin x = \int 2\cos x\sin x\,dx + C$$

$\Rightarrow \quad y^2\sin x = \sin^2 x + C$

When $\quad x = \frac{\pi}{2},\ y = 1$

$$1 = 1 + C \Rightarrow C = 0$$

$\therefore \quad y^2 = \sin x$

6. (c) We know,

Root mean square $\geq$ Arithmetic mean

$$\therefore \quad \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}} \geq \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\Rightarrow \quad \sqrt{\frac{400}{n}} \geq \frac{80}{n}$$

$$\Rightarrow \quad \frac{400}{n} - \frac{6400}{n^2} \geq 0$$

$$\Rightarrow \quad \frac{400}{n^2}(n-16) \geq 0 \Rightarrow n \geq 16$$

7. (b) Given, $p = 2q$

$$\because \quad p + q = 1$$

$$\Rightarrow \quad p = \frac{2}{3}$$

and $\quad q = \frac{1}{3}$

$\therefore$ Required probability

$$= {}^4C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 + {}^4C_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^1 + {}^4C_4\left(\frac{2}{3}\right)^4$$

$$= 6 \times \frac{4}{81} + \frac{4 \times 8}{81} + \frac{1 \times 16}{81} = \frac{72}{81} = \frac{8}{9}$$

8. (b) $\sin(3 \sin^{-1}(0.8)) = \sin[3(0.8) - (0.8)^3]$

$= \sin[2.4 - 0.512] = \sin(1.88)$

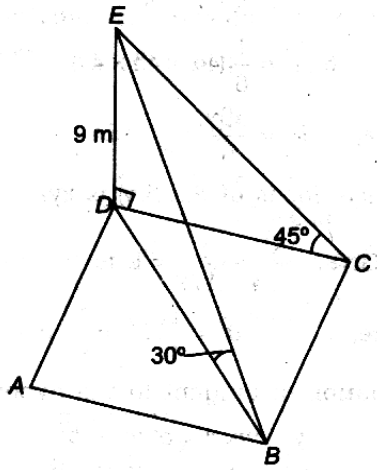
9. (a) Let $ABCD$ be a rectangle and DE be the vertical lamp post.

In ΔECD, $\tan 45° = \frac{9}{DC}$

$\Rightarrow$ $DC = 9$ m

Diagonal $BD = 9 \cot 30° = 9\sqrt{3}$ m

In ΔABD, $AD = \sqrt{(9\sqrt{3})^2 - 9^2} = 9\sqrt{2}$ m



$\therefore$ Area of rectangle $ABCD = 9 \times 9\sqrt{2}$

$= 81\sqrt{2}$ m^2

10. (c) Since, $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = |\mathbf{d}| = 1$

and $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = 0$

$\therefore$ $|\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}|^2 = \Sigma |\mathbf{a}|^2 + 2\Sigma \mathbf{a} \cdot \mathbf{b}$

$= 4 + 2[\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a} + \mathbf{d} \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c})]$

$= 4 + 2\,\mathbf{d} \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c})$

Let $\mathbf{d} = \lambda \mathbf{a} + \mu \mathbf{b} + \nu \mathbf{c}$

Then, $\mathbf{d} \cdot \mathbf{a} = \mathbf{d} \cdot \mathbf{b} = \mathbf{d} \cdot \mathbf{c}$

$\Rightarrow$ $\lambda = \mu = \nu = \cos\theta$

Also, $\lambda^2 + \mu^2 + \nu^2 = 1$

$\Rightarrow$ $3 \cos^2 \theta = 1$

$\Rightarrow$ $\cos\theta = \pm \frac{1}{\sqrt{3}}$

$\therefore$ $|\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}|^2 = 4 \pm \frac{2 \cdot 3}{\sqrt{3}} = 4 \pm 2\sqrt{3}$

11. (d) Let the variable plane intersects the coordinate axes at $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$. Then, equation of plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Let $P(\alpha, \beta, \gamma)$ be the centroid of tetrahedron $OABC$, then

$$\alpha = \frac{a}{4}, \beta = \frac{b}{4}, \gamma = \frac{c}{4}$$

$\Rightarrow$ $a = 4\alpha, b = 4\beta, c = 4\gamma$

$\therefore$ Volume of tetrahedron

$= \frac{1}{3}$ (Area of $\Delta AOB)OC$

$\Rightarrow$ $64k^3 = \frac{1}{3}\left(\frac{1}{2} ab\right)c$

$\Rightarrow$ $64k^3 = \frac{1}{6}(4\alpha \times 4\beta \times 4\gamma)$

$\Rightarrow$ $k^3 = \frac{\alpha\beta\gamma}{6}$

Hence, locus of $P(\alpha, \beta, \gamma)$ is $xyz = 6k^3$

12. (c) Given, $\frac{x^2}{1/4} - \frac{y^2}{1/9} = 1$

Here, $a^2 = \frac{1}{4}, b^2 = \frac{1}{9}$

Equation of tangent to the hyperbola is

$$y = mx \pm \sqrt{a^2 m^2 - b^2}$$

$$= mx \pm \sqrt{\frac{m^2}{4} - \frac{1}{9}}$$

Slope of given line is $m = \frac{5}{4}$

$\therefore$ $y = \frac{5}{4} x \pm \sqrt{\frac{25}{64} - \frac{1}{9}}$

$\Rightarrow$ $24y - 30x = \pm \sqrt{161}$

13. (b) The equation of the auxiliary circle is $x^2 + y^2 = a^2$. Let (h, k) be the pole, then equation of the polar of (h, k) with respect to the given ellipse is

$$\frac{hx}{a^2} + \frac{ky}{b^2} = 1$$

Since, this is tangent to the circle.

$$\frac{|0 + 0 - 1|}{\sqrt{\left(\frac{h}{a^2}\right)^2 + \left(\frac{k}{b^2}\right)^2}} = \pm a$$

$\Rightarrow$ $\frac{h^2}{a^4} + \frac{k^2}{b^4} = \frac{1}{a^2}$

Hence, locus of (h, k) is

$$\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2}$$

14. (a) Given inequality can be rewritten as

$$\frac{(2x + 3)(3x - 4)^3 (x - 4)}{(x - 2)^2 x^5} \geq 0$$

or $(2x + 3)(3x - 4)^3 (x - 4)(x - 2)^2$

$x^5 \geq 0, x \neq 0, 2$

−	−	−	+	+	+
+	+	+	+	+	+
−	−	−	−	−	+
−	−	+	+	+	+
−	+	+	+	+	+

$-\frac{3}{2}$ 0 $-\frac{3}{2}$ 2 4

In a sign scheme when we multiply the sign in each interval, we get the positive inteval.

$$\left(-\infty, \frac{-3}{2}\right) \cup \left(0, \frac{4}{3}\right) \cup (4, \infty)$$

15. (d) Since, focus $(0, 1)$ and directrix is $x + 2 = 0$

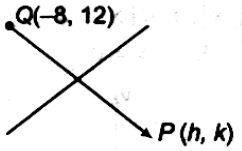
$$\therefore \quad (x-0)^2 + (y-1)^2 = \left(\frac{x+2}{\sqrt{1}}\right)^2$$

$$\Rightarrow \quad (y-1)^2 = 4(x+1)$$

$$\therefore \quad x = t^2 - 1 \text{ and } y = 2t + 1$$

16. (b) Centre and radius of given circle is $(-8, 12)$ and $r = \sqrt{64 + 144 - 183} = 5$

Let $P(h, k)$ be the image of the point $Q(-8, 12)$ with respect to the line mirror.



$\therefore$ Mid-point of $PQ \left(\left(\frac{h-8}{2}\right), \left(\frac{k+12}{2}\right)\right)$ lies on the line $4x + 7y + 13 = 0$

$$\therefore \quad 4\left(\frac{h-8}{2}\right) + 7\left(\frac{k+12}{2}\right) + 13 = 0$$

$$\Rightarrow \quad 4h + 7k + 78 = 0 \quad \ldots(i)$$

Also, PQ is perpendicular to $4x + 7y + 13 = 0$.

$$\therefore \quad \frac{k-12}{h+8} \times \frac{4}{7} = -1$$

$$\Rightarrow \quad 7h - 4k + 104 = 0 \quad \ldots(ii)$$

On solving Eqs. (i) and (ii), we get

$$(-16, -2)$$

The equation of the required circle is

$$(x+16)^2 + (y+2)^2 = 5^2$$

$$\Rightarrow \quad x^2 + y^2 + 32x + 4y + 235 = 0$$

17. (c) According to the given condition

Area $(\Delta POA) = 2$ area (ΔPOB)

Let $P(h, k)$ be any point such that

$$\left|\frac{1}{2} 4h\right| = 2\left|\frac{1}{2} 6k\right|$$

$$\Rightarrow \quad |4h| = |12k|$$

$$\Rightarrow \quad |h| = |3k| \Rightarrow h = \pm 3k$$

$$\Rightarrow \quad (h + 3k)(h - 3k) = 0$$

Hence, locus of a point is $(x + 3y)(x - 3y) = 0$.

18. (a) Given equation is

$$x^3 + 3ax^2 + 3bx + c = 0$$

$$\therefore \quad z_1 + z_2 + z_3 = -3a$$

$$\Rightarrow \quad \frac{z_1 + z_2 + z_3}{3} = -a$$

and $\quad z_1z_2 + z_2z_3 + z_3z_1 = 3b$

For equilateral triangle,

$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$$

$$\Rightarrow \quad (z_1 + z_2 + z_3)^2 = 3(z_1z_2 + z_2z_3 + z_3z_1)$$

$$\Rightarrow \quad (-3a)^2 = 3(3b) \Rightarrow a^2 = b$$

19. (a) Since, a, b and c are in GP.

$\therefore \log a, \log b$ and $\log c$ are in AP.

$$\Rightarrow \quad 2 \log b = \log a + \log c \quad \ldots(i)$$

Given, $\quad \sqrt[x]{a} = \sqrt[y]{b} = \sqrt[z]{c}$

$$\Rightarrow \quad a^{1/x} = b^{1/y} = c^{1/z}$$

$$\Rightarrow \quad \frac{1}{x} \log a = \frac{1}{y} \log b = \frac{1}{z} \log c = k \text{ (say)}$$

$$\therefore \quad \log a = kx, \log b = ky, \log c = kz \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$2ky = kx + kz, k \neq 0 \Rightarrow 2y = x + z$$

20. (d) It is equal to, 30 mangoes can be distributed among 4 boys such that each boy can receive any number of mangoes.

$\therefore$ Total number of ways $= {}^{30+4-1}C_{4-1}$

$$= {}^{33}C_3 = \frac{33 \cdot 32 \cdot 31}{1 \cdot 2 \cdot 3} = 5456$$

21. (b) Given, $(1 + x)^n = C_0 + C_1x + \ldots + C_nx^n$

Integrating both sides, we get

$$\int_0^x (1+x)^n \, dx = \int_0^x \left(C_0 + C_1 x + C_2 \frac{x^2}{2} + \ldots + C_n x^n\right) dx$$

$$\Rightarrow \quad \left[\frac{(1+x)^{n+1}}{n+1}\right]_0^x = C_0x + C_1 \frac{x^2}{2} + C_2 \frac{x^3}{3} + \ldots + C_n \frac{x^{n+1}}{n+1}$$

$$\Rightarrow \quad \frac{(1+x)^{n+1} - 1}{1+n} = C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + \ldots + \frac{C_nx^{n+1}}{n+1} \quad \ldots(i)$$

and $\quad (x+1)^n = C_0x^n + C_1x^{n-1} + C_2x^{n-2} + \ldots + C_n \quad \ldots(ii)$

Multiplying Eqs. (i) and (ii), we get

$$\frac{1}{(n+1)}\{(1+x)^{2n+1} - (1+x)^n\} = \left(C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + \ldots + \frac{C_nx^{n+1}}{n+1}\right) \times (C_0x^n + C_1x^{n-1} + \ldots + C_n)$$

Coefficient of x^{n+1} on both sides, we get

$$\frac{1}{(n+1)}\{{}^{2n+1}C_{n+1} - 0\} = C_0^2 + \frac{C_1^2}{2} + \ldots + \frac{C_n^2}{(n+1)}$$

$$\Rightarrow \quad \frac{(2n+1)!}{(n+1) \cdot (n+1)!\, n!} = C_0^2 + \frac{C_1^2}{2} + \ldots + \frac{C_n^2}{(n+1)}$$

22. (a) Let a be the first term and d be the common difference of an AP.

$$\therefore \quad T_p = a + (p-1)d,$$

$$T_{2q} = a + (2q-1)d,$$

$$T_{3r} = a + (3r-1)d$$

$$\therefore \quad \Delta = xyz \begin{vmatrix} 1/x & 1/y & 1/z \\ p & 2q & 3r \\ 1 & 1 & 1 \end{vmatrix}$$

$$= xyz \begin{vmatrix} a+(p-1)d & a+(2q-1)d & a+(3r-1)d \\ p & 2q & 3r \\ 1 & 1 & 1 \end{vmatrix}$$

Applying $R_1 \to R_1 - aR_3$, $R_2 \to R_2 - R_3$ and then taking d common from R_1, we get

$$\Delta = xyzd \begin{vmatrix} (p-1) & (2q-1) & (3r-1) \\ (p-1) & (2q-1) & (3r-1) \\ 1 & 1 & 1 \end{vmatrix} = 0$$

23. (c) Given, $f'(0) = 3$

$$\Rightarrow \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = 3$$

$$\Rightarrow \frac{f(0+h) - f(0)}{h} = 3$$

$$\Rightarrow \frac{f(0)\, f(h) - f(0)}{h} = 3$$

$$\Rightarrow f(0) \left(\lim_{h \to 0} \frac{f(h) - 1}{h} \right) = 3 \quad \ldots(i)$$

Also, $f(x + y) = f(x)\, f(y)$

$$\Rightarrow f(0) = f(0)\, f(0)$$

$$\Rightarrow f(0)\,[1 - f(0)] = 0 \Rightarrow f(0) = 1$$

Put $f(0) = 1$ in Eq. (i), we get

$$\lim_{h \to 0} \frac{f(h) - 1}{h} = 3 \quad \ldots(ii)$$

Now, $f'(5) = \lim_{h \to 0} \frac{f(5+h) - f(h)}{h}$

$$= \lim_{h \to 0} \frac{f(5)\, f(h) - f(h)}{h}$$

$$= f(5) \left(\lim_{h \to 0} \frac{f(h) - 1}{h} \right)$$

$$= 2 \times 3 = 6 \quad \text{[from Eq. (ii)]}$$

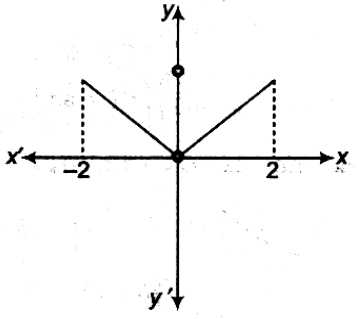
24. (b) $y = \cos^{-1}(\cos x)$

$$= \cos^{-1}(\cos(\pi - \overline{x - \pi}))$$

$$y = 2\pi - x$$

$$\Rightarrow \frac{dy}{dx} = -1$$

25. (a) $f(x) = \begin{cases} |x|, & 0 < |x| \le 2 \\ 1, & x = 0 \end{cases}$



It is clear from the graph $f(x)$ has point of local maximum at $x = 0$.

26. (b) $\sin x = x^2 + x + 1$

Let $y = \sin x$,

then $y = x^2 + x + 1$

Since, $\sin x$ is real.

But $x^2 + x + 1 = (x - \omega)(x - \omega^2)$

It means roots are imaginary.

Hence, no solution exist.

Also, Statement II is true but it is not a correct explanation for Statement I.

27. (a) $\lim_{x \to 0^+} \frac{x}{a} \left[\frac{b}{x} \right] = \lim_{h \to 0} \frac{h}{a} \left[\frac{b}{h} \right]$

$$= \lim_{h \to 0} \frac{h}{a} \times \frac{b}{h} = \frac{b}{a}$$

Also, $\lim_{x \to \infty} \frac{\{x\}}{x} \to 0$

28. (d) $a_{ij} = \frac{i - j}{i + 2j}$

$$\therefore A = \begin{bmatrix} 0 & -\frac{1}{5} & -\frac{2}{7} \\ \frac{1}{4} & 0 & -\frac{1}{8} \\ \frac{2}{5} & \frac{1}{7} & 0 \end{bmatrix}$$

which is neither symmetric nor skew-symmetric. But every matrix can be expressed as a sum of symmetric and skew-symmetric matrix.

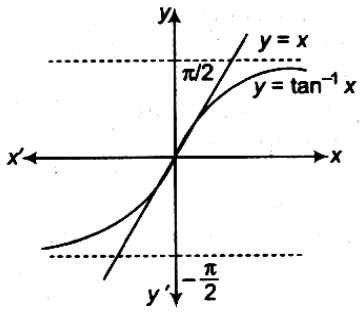
29. (a) It is clear from the graph that

$$\tan^{-1} x < x \text{ when } x > 0$$

$$\Rightarrow \frac{\tan^{-1} x}{x} < 1$$

For $x < 0$,

$$\tan^{-1} x > x$$



$$\Rightarrow \frac{\tan^{-1} x}{x} < 1$$

$$\therefore \lim_{x \to 0} \left[\frac{\tan^{-1} x}{x} \right] = 0$$

30. (c) Let $f(x) = x \sin x - \frac{1}{2} \sin^2 x$

$$f'(x) = x \cos x + \sin x - \sin x \cos x$$

$$= \sin x\,(1 - \cos x) + x \cos x$$

For $x \in \left(0, \frac{\pi}{2}\right)$, $\sin x > 0$, $(1 - \cos x) > 0$,

$$\cos x > 0$$

$$\Rightarrow f'(x) > 0, \forall\, x \in \left(0, \frac{\pi}{2}\right)$$

$\Rightarrow f(x)$ is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

Now, $\lim_{x\to 0} f(x) = 0$

and $\lim_{x\to \frac{\pi}{2}} f(x) = \frac{\pi - 1}{2}$

$\therefore$ Range of $f(x) = \left(0, \frac{\pi - 1}{2}\right)$

31. (c) **32.** (a) **33.** (c)

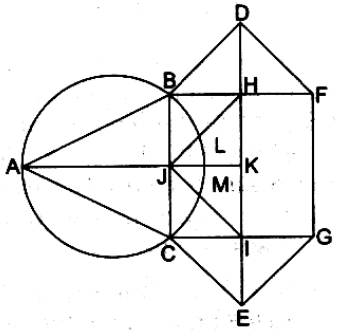
34. (a) In all other figures, one of the dots lies outside the triangle as well as the circle.

35. (c) In fig. (c), the arrow indicates an anti-clockwise motion while in each of the other figures, the arrow indicates a clockwise motion.

36. (d) Both the larger and the smaller squares move to the adjacent corner anti-clockwise in each turn. Also, the shading in the smaller square moves 1, 2, 3, 4, 5, ... steps anti-clockwise sequentially and the shading in the larger square moves 1, 2, 3, 4, 5,.... steps clockwise sequentially.
In all other figures, the number of dots outside the main figure is one more than the number of dots inside the main figure.

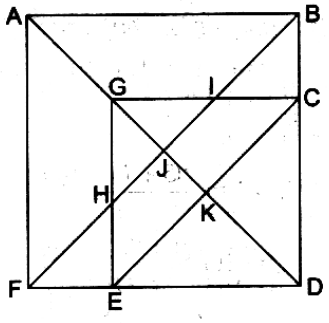
37. (b) **38.** (a) **39.** (b) **40.** (c) **41.** (d)

42. (d) The figure may be labelled as shown as below



The simplest triangles are ABJ, ACJ, BDH, DHF, CIE and GIE *i.e.*, 6 in number.
The triangles composed of two components are ABC, BDF, CEG, BHJ, JHK, JKI and CJI *i.e.*, 7 in number.
There is only one triangle JHI which is composed of four components.
Thus, there are 6 + 7 + 1 = 14 triangles in the given figure.

43. (a) The figure may be labelled as shown as below



The simplest triangles are IJO, BCJ, CDK, KQL, MLQ, GFM, GHN and NIO *i.e.*, 8 in number.

The triangles composed of two components are ABO, AHO, NIJ, IGP, ICP, DEQ, FEQ. The simplest triangles are EFH, BIC, GHJ, GIJ, EKD and CKD *i.e.*, 6 in number.
The triangles composed of two components are ABJ, AFJ, GCK, GEK, CED and GHI *i.e.*, 6 in number.

The triangles composed of three components are GCD, GED, DJB and DJF *i.e.*, 4 in number.

The triangles composed of four components are ABF and GCE *i.e.*, 2 in number.

The triangles composed of five components are ABD and AFD *i.e.*, 2 in number.

There is only one triangle *i.e.*, FBD composed of six components.

Total number of triangles in the figure
= 6 + 6 + 4 + 2 + 2 + 1 = 21

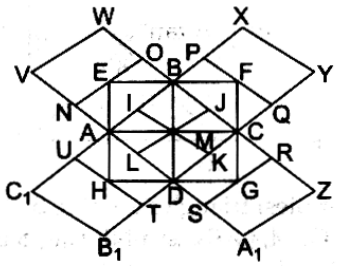
44. (a) Both the larger and the smaller squares move to the adjacent corner anti-clockwise in each turn. Also, the shading in the smaller square moves 1, 2, 3, 4, 5, ... steps anti-clockwise sequentially and the shading in the larger square moves 1, 2, 3, 4, 5,.... steps clockwise sequentially.

45. (c) In each step, one line segment is lost from the clockwise end of the outer element and a new line segment appears at the anti-clockwise end. Also, the inner 'L' shaped element rotates 90° clockwise in each step.

46. (d) In each step, the first element moves to the third position and gets replaced by a new element; the second and the third elements move to the first and the second positions respectively and the entire figure rotates 90° clockwise.

47. (d) In one step, the existing element enlarges and a new element appears inside this element. In the next step, the outer element is lost.

48. (d) The figure may be labelled as shown as below



The squares composed of two components are BJMI, CKMJ, DLMK and AIML *i.e.*, 4 in number.

The squares composed of three components are EBMA, BFCM, MCGD and AMDH *i.e.*, 4 in number.

The squares composed of four components are VWBA, XYCB, ZA_1DC and B_1C_1AD *i.e.*, 4 in number.

The squares composed of seven components are NOJL, PQKI, RSLJ and TUIK *i.e.*, 4 in number.

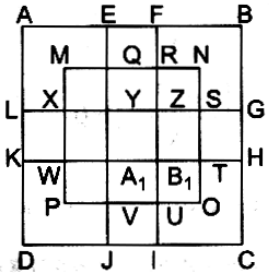
There is only one square *i.e.*, ABCD composed of eight components.

There is only one square i.e. EFGH composed of twelve components.

Total number of squares in the figure

$= 4 + 4 + 4 + 4 + 1 + 1 = 18$

49. (a) The figure may be labelled as shown as below



The simplest squares are EFRQ, MQYX, QRZY, RNSZ, LXWK, XYA$_1$W, YZB$_1$A$_1$, ZSTB$_1$, SGHT, WA$_1$VP, A$_1$B$_1$UV, B$_1$TOU and VUIJ *i.e.*, 13 in number.

The squares having two components, are AEYL, FBGZ, KA$_1$JD and B$_1$HCI *i.e.*, 4 in number.

The squares having four components are MRB$_1$W, QNTA$_1$ XZUP and YSOV *i.e.*, 4 in number.

The squares having seven components each are AFB$_1$K, EBHA$_1$ LZID and YGCJ *i.e.*, 4 in number.

There is only one square *i.e.*, MNOP composed of nine components.

There is only one square *i.e.*, ABCD composed of seventeen components.

There are $13 + 4 + 4 + 4 + 1 + 1 = 27$ squares in the figure.

50. (b) **51.** (c) **52.** (a) **53.** (d) **54.** (a)

55. (d) **56.** (b) **57.** (c) **58.** (b) **59.** (d)

60. (d) **61.** (a) **62.** (c) **63.** (c) **64.** (b)

65. (d) **66.** (b) **67.** (c) **68.** (a) **69.** (a)

70. (b) **71.** (d) **72.** (a) **73.** (a) **74.** (d)

75. (c) **76.** (d) **77.** (a) **78.** (a) **79.** (a)

80. (a)