10. Let $a_1, a_2, a_3, \ldots$ be an arithmetic progression with $a_1 = 7$ and common difference 8. Let $T_1, T_2, T_3, \ldots$ be such that $T_1 = 3$ and $T_{n+1} - T_n = a_n$ for $n \ge 1$. Then, which of the following is/are TRUE ?

(A) $T_{20} = 1604$

(B) $\sum_{k=1}^{20} T_k = 10510$

(C) $T_{30} = 3454$

(D) $\sum_{k=1}^{30} T_k = 35610$

Ans. (B,C)

Sol. $a_1 = 7, d = 8$

$T_{n+1} - T_n = a_n \; \forall n \geq 1$

$S_n = T_1 + T_2 + T_3 + \ldots + T_{n-1} + T_n$

$S_n = T_1 + T_2 + T_3 + \ldots + T_{n-1} + T_n$

on subtraction

$T_n = T_1 + a_1 + a_2 + \ldots + a_{n-1}$

$T_n = 3 + (n - 1)(4n - 1)$

$T_n = 4n^2 - 5n + 4$

$$\sum_{k=1}^{n} T_k = 4\sum n^2 - 5\sum n + 4n$$

$T_{20} = 1504$

$T_{30} = 3454$

$$\sum_{k=1}^{30} T_k = 35615$$

$$\sum_{k=1}^{20} T_k = 10510$$

Sol. $\vec{r} = \hat{k} + t(-\hat{i} + \hat{j}) + p(-\hat{i} + \hat{k})$

$\vec{n} = \hat{i} + \hat{j} + \hat{k}$

$\Rightarrow x + y + z = 1$

Q(10,15,20) and S(α, β, γ)

$$\frac{\alpha - 10}{1} = \frac{\beta - 15}{1} = \frac{\gamma - 20}{1} = -2\left(\frac{10 + 15 + 20 - 1}{1 + 1 + 1}\right)$$

$$= -\frac{88}{3}$$

$$\Rightarrow (\alpha, \beta, \gamma) \equiv \left(-\frac{58}{3}, -\frac{43}{3}, -\frac{28}{3}\right)$$

$\Rightarrow$ A,B,C are correct options

13. Consider the parabola $y^2 = 4x$. Let S be the focus of the parabola. A pair of tangents drawn to the parabola from the point P = (–2, 1) meet the parabola at P_1 and P_2. Let Q_1 and Q_2 be points on the lines SP_1 and SP_2 respectively such that PQ_1 is perpendicular to SP_1 and PQ_2 is perpendicular to SP_2. Then, which of the following is/are TRUE ?

(A) *SQ1* = 2

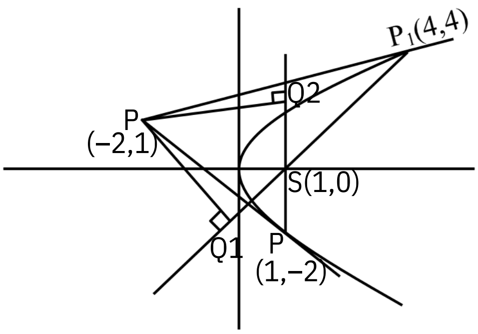
(B) Q1Q2= $\frac{3\sqrt{10}}{5}$

(C) PQ1 = 3

(D) SQ2 = 1

Ans. (B,C,D)

Sol. Let equation of tangent with slope 'm' be



$$T : y = mx + \frac{1}{m}$$

T : passes through (–2, 1) so

$$1 = -2m + \frac{1}{m}$$

$\Rightarrow$ $m = -1$ or $m = \frac{1}{2}$

Points are given by $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

So, one point will be (1, −2) & (4, 4)

Let $P_1(4, 4)$ & $P_2(1, -2)$

$P_1S : 4x - 3y - 4 = 0$

$P_2S : x - 1 = 0$

$PQ_1 = \left|\frac{4(-2) - 3(1) - 4}{5}\right| = 3$

$SP = \sqrt{10}$; $PQ_2 = 3$; $SQ_1 = 1 = SQ_2$

$\frac{1}{2}\left(\frac{Q_1Q_2}{2}\right) \times \sqrt{10} = \frac{1}{2} \times 3 \times 1$ (comparing Areas)

$\Rightarrow Q_1Q_2 = \frac{2 \times 3}{\sqrt{10}} = \frac{3\sqrt{10}}{5}$

14. Let $|M|$ denote the determinant of a square matrix M. Let $g : \left[0, \frac{\pi}{2}\right] \rightarrow \mathbb{R}$ be the function defined by

$$g(\theta) = \sqrt{f(\theta) - 1} + \sqrt{f\left(\frac{\pi}{2} - \theta\right) - 1}$$

where

$$f(\theta) = \frac{1}{2}\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} + \begin{vmatrix} \sin\pi & \cos\left(\theta + \frac{\pi}{4}\right) & \tan\left(\theta - \frac{\pi}{4}\right) \\ \sin\left(\theta - \frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right) \\ \cot\left(\theta + \frac{\pi}{4}\right) & \log_e\left(\frac{\pi}{4}\right) & \tan\pi \end{vmatrix}.$$

Let $p(x)$ be a quadratic polynomial whose roots are the maximum and minimum values of the function $g(\theta)$, and $p(2) = 2 - \sqrt{2}$. Then, which of the following is/are TRUE ?

(A) $p\left(\frac{3 + \sqrt{2}}{4}\right) < 0$

(B) $p\left(\frac{1 + 3\sqrt{2}}{4}\right) > 0$

(C) $p\left(\frac{5\sqrt{2} - 1}{4}\right) > 0$

(D) $p\left(\frac{5 - \sqrt{2}}{4}\right) < 0$

Ans. (A,C)

Sol. $f(\theta)=\frac{1}{2}\begin{vmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix}+\begin{vmatrix}\sin\pi & \cos\left(\theta+\frac{\pi}{4}\right) & \tan\left(\theta-\frac{\pi}{4}\right)\\ \sin\left(\theta-\frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right)\\ \cot\left(\theta+\frac{\pi}{4}\right) & \log_e\frac{\pi}{4} & \tan\pi\end{vmatrix}$

$f(\theta)=\frac{1}{2}\begin{vmatrix}2 & \sin\theta & 1\\ 0 & 1 & \sin\theta\\ 0 & -\sin\theta & 1\end{vmatrix}+\begin{vmatrix}0 & -\sin\left(\theta-\frac{\pi}{4}\right) & \tan\left(\theta-\frac{\pi}{4}\right)\\ \sin\left(\theta-\frac{\pi}{4}\right) & 0 & \log_e\left(\frac{4}{\pi}\right)\\ -\tan\left(\theta-\frac{\pi}{4}\right) & -\log_e\left(\frac{4}{\pi}\right) & 0\end{vmatrix}$

$f(\theta) = (1+\sin^2\theta) + 0$ (skew symmetric)

$g(\theta)=\sqrt{f(\theta)-1}+\sqrt{f\left(\frac{\pi}{2}-\theta\right)-1}$

$= |\sin\theta| + |\cos\theta|$ for $\theta\in\left[0,\frac{\pi}{2}\right]$

$g(\theta)\in\left[1,\sqrt{2}\right]$

Again let $P(x)=k\left(x-\sqrt{2}\right)(x-1)$

$2-\sqrt{2}=k\left(2-\sqrt{2}\right)(2-1)$

$\Rightarrow k=1$ $\left(P(2)=2-\sqrt{2}\text{ given}\right)$

$\therefore P(x)=\left(x-\sqrt{2}\right)(x-1)$

for option (A) $P\left(\frac{3+\sqrt{2}}{4}\right)<0$ correct

option (B) $P\left(\frac{1+3\sqrt{2}}{4}\right)<0$ incorrect

option (C) $P\left(\frac{5\sqrt{2}-1}{4}\right)>0$ correct

option (D) $P\left(\frac{5-\sqrt{2}}{4}\right)>0$ incorrect

SECTION-3 : (Maximum Marks : 12)

This section contains **FOUR (04)** Matching List Sets.
Each set has **ONE** Multiple Choice Question.
Each set has **TWO** lists : **List-I** and **List-II**.
List-I has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Five** entries (P), (Q), (R), (S) and (T).
FOUR options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 In all other cases.

15. Consider the following lists:

List-I		List-II	
(I)	$\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(P)	has two elements
(II)	$\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(Q)	has three elements
(III)	$\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	(R)	has four elements
(IV)	$\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S)	has five elements
		(T)	has six elements

The correct option is:

(A) (I) → (P); (II) → (S); (III) → (P); (IV) → (S)

(B) (I) → (P); (II) → (P); (III) → (T); (IV) → (R)

(C) (I) → (Q); (II) → (P); (III) → (T); (IV) → (S)

(D) (I) → (Q); (II) → (S); (III) → (P); (IV) → (R)

Ans. (B)

Sol. (I) $\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$

$\cos x + \sin x = 1$

$\Rightarrow \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos\left(x-\frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

$\Rightarrow x-\frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$; $n \in Z$

$\Rightarrow x = 2n\pi$; $x = 2n\pi + \frac{\pi}{2}$; $n \in Z$

$\Rightarrow x \in \left\{0, \frac{\pi}{2}\right\}$ in given range has two solutions

(II) $\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$

$\sqrt{3}\tan 3x = 1 \Rightarrow \tan 3x = \frac{1}{\sqrt{3}} \Rightarrow 3x = n\pi + \frac{\pi}{6}$

$\Rightarrow x = (6n+1)\frac{\pi}{18}$; $n \in Z$

$\Rightarrow x \in \left\{\frac{\pi}{18}, -\frac{5\pi}{18}\right\}$ in given range has two solutions

(III) $\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$

$2\cos 2x = \sqrt{3}$

$\Rightarrow \cos 2x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\Rightarrow 2x = 2n\pi \pm \frac{\pi}{6}$; $n \in Z$

$\Rightarrow x = n\pi \pm \frac{\pi}{12}$; $n \in Z$

$x \in \left\{\pm\frac{\pi}{12}, \pi \pm \frac{\pi}{12}, -\pi \pm \frac{\pi}{12}\right\}$

Six solutions in given range

(IV) $\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$

$\cos x - \sin x = -1$

$\Rightarrow \cos\left(x+\frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}} = \cos\frac{3\pi}{4}$; $n \in Z$

$\Rightarrow x + \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4}$

$\Rightarrow x = 2n\pi + \frac{\pi}{2}$ or $x = 2n\pi - \pi$; $n \in Z$

$\Rightarrow x \in \left\{-\frac{3\pi}{2}, \frac{\pi}{2}, \pi, -\pi\right\}$ four solutions in given range

17. Let , V, ݣ be nonzero real numbers that are, respectively, the 10 hپ, 100 hپ and 1000 hپ terms of a harmonic progression. Consider the system of linear equations

1 = ي + ٻ + ٽ

10ي + 100ٻ + 1000 0 = ٽ

.0 = Vݣ ي + ݣ ٻ + V ٽ

List-I		List-II	
(I)	If $\frac{q}{r} = 10$, then the system of linear equations has	(P)	$x = 0, y = \frac{10}{9}, z = -\frac{1}{9}$ as a solution
(II)	If $\frac{p}{r} \neq 100$, then the system of linear equations has	(Q)	$x = \frac{10}{9}, y = -\frac{1}{9}, z = 0$ as a solution
(III)	If $\frac{p}{q} \neq 10$, then the system of linear equations has	(R)	infinitely many solutions
(IV)	If $\frac{p}{q} = 10$, then the system of linear equations has	(S)	no solution
		(T)	at least one solution

The correct option is:

(A) (I) → (T); (II) → (R); (III) → (S); (IV) → (T)

(B) (I) → (Q); (II) → (S); (III) → (S); (IV) → (R)

(C) (I) → (Q); (II) → (R); (III) → (P); (IV) → (R)

(D) (I) → (T); (II) → (S); (III) → (P); (IV) → (T)

Ans. (B)

Sol. If $\frac{q}{r} = 10 \Rightarrow \Delta = D \Rightarrow D_x = D_y = D_z = 0$

So, there are infinitely many solutions

Look of infinitely many solutions can be given as

$x + y + z = 1$

& $10x + 100y + 1000z = 0 \Rightarrow \quad x + 10y + 100z = 0$

Let $z = \lambda$

then $x + y = 1 - \lambda$

and $x + 10y = -100\lambda$

$\Rightarrow x = \frac{10}{9} + 10\lambda$; $y = \frac{-1}{9} - 11\lambda$

i.e., $(x, y, z) \equiv \left(\frac{10}{9} + 10\lambda, \frac{-1}{9} - 11\lambda, \lambda\right)$

$Q\left(\frac{10}{9}, \frac{-1}{9}, 0\right)$ valid for $\lambda = 0$

$P\left(0, \frac{10}{9}, \frac{-1}{9}\right)$ not valid for any λ.

(I) $\rightarrow$ Q,R,T

(II) If $\frac{p}{r} \neq 100$, then $D_y \neq 0$

So no solution

(II) $\rightarrow$ (S)

(III) If $\frac{p}{q} \neq 10$, then $D_z \neq 0$ so, no solution

(III) $\rightarrow$ (S)

(IV) If $\frac{p}{q} = 10 \Rightarrow D_z = 0 \Rightarrow D_x = D_y = 0$

so infinitely many solution

(IV) $\rightarrow$ Q,R,T

18. Consider the ellipse

$$\frac{x2}{4}+\frac{y2}{3}=1.$$

Let H(,2 > , 0), 0 < be a point. A straight line drawn through ذ parallel to theب- axis crosses the ellipse and its auxiliary circle at points ے and ي respectively, in the first quadrant. The tangent to the ellipse at the point ے intersects the positiveب-axis at a point م. Suppose the straight line joining ي and the origin makes an angle f with the positiveب-axis .

List-I		List-II	
(I)	If $f = \frac{p}{4}$, then the area of the triangle ذمي is	(P)	$\frac{(\sqrt{3}-1)^4}{8}$
(II)	If f $\frac{p=}{3}$, then the area of the triangle ذمي is	(Q)	1
(III)	If f $\frac{p=}{6}$, then the area of the triangle ذمي is	(R)	$\frac{3}{4}$
(IV)	If f $\frac{p=}{12}$, then the area of the triangle is ذمي	(S)	$\frac{1}{2\sqrt{3}}$
		(T)	$\frac{3\sqrt{3}}{2}$

The correct option is: (A) (I) →

(R); (II) → (S); (III) (B) (I) →→ (Q); (IV) → (P)

(R); (II) → (T); (III) → (S); (IV) → (P)

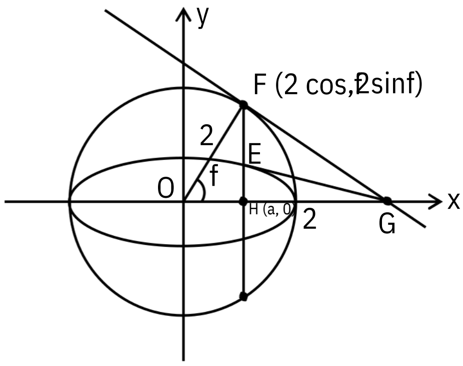
(C) (I) → (Q); (II) → (T); (III) → (S); (IV) → (P)

(D) (I) → (Q); (II) → (S); (III) → (Q); (IV) → (P)

Ans. (C)

Sol. Let $F(2\cos\phi, 2\sin\phi)$

$\&\ E(2\cos\phi, \sqrt{3}\sin\phi)$



$$EG : \frac{x}{2}\cos\phi + \frac{y}{\sqrt{3}}\sin\phi = 1$$

$$\therefore\ G\left(\frac{2}{\cos\phi}, 0\right) \text{ and } \alpha = 2\cos\phi$$

$$ar(\Delta FGH) = \frac{1}{2}\ HG \times FH$$

$$= \frac{1}{2}\left(\frac{2}{\cos\phi} - 2\cos\phi\right) \times 2\sin\phi$$

$$f(\phi) = 2\tan\phi\sin^2\phi$$

$$\therefore\ \text{(I)}\ f\left(\frac{\pi}{4}\right) = 1 \qquad \text{(II)}\ f\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{2} \qquad \text{(III)}\ f\left(\frac{\pi}{6}\right) = \frac{1}{2\sqrt{3}}$$

$$\text{(IV)}\ f\left(\frac{\pi}{12}\right) = 2\left(2-\sqrt{3}\right)\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2 = \frac{\left(4-2\sqrt{3}\right)\left(\sqrt{3}-1\right)^2}{8} = \frac{\left(\sqrt{3}-1\right)^4}{8}$$

$\therefore$ (I) $\rightarrow$ (Q) ; (II) $\rightarrow$ (T) ; (III) $\rightarrow$ (S) ; (IV) $\rightarrow$ (P)